# *BEYOND EXPECTED VALUE: MAKING DECISIONS UNDER RISK AND UNCERTAINTY*



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# *BEYOND EXPECTED VALUE: MAKING DECISIONS UNDER RISK AND UNCERTAINTY*


by

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# EXECUTIVE SUMMARY

Risk is a characteristic of a situation in which a number of outcomes are possible, the particular one that will occur is uncertain and at least one of the possibilities is undesirable. Much of the work of the U.S. Army Corps of Engineers (Corps) is involved with managing the risk associated with natural processes, such as flooding and drought. Safety and quality of human life, large amounts of money and environmental issues are all at stake in the decisions made by the Corps relating to the management of natural risks.

Risk analysis is encouraged by regulation and guidance as a “way of doing business” within the Corps and is increasingly used in technical aspects of plan formulation. Risk analysis is comprised of three components—risk assessment (analysis of the technical aspects of the problem to determine uncertainties and their magnitudes), risk communication (conveying information about the nature of the risks to all interested parties) and risk management (deciding how to handle risks). The majority of theCorps efforts have been devoted to the technical factors of risk assessment, providing quantitative and qualitative estimates of the results of alternatives, containing measures of the risk associated with those alternatives. Less attention has been paid to the issues of communication and management, in particular with respect to the role of decision-making in the presence of risk information.

Typical practices within the Corps where risk assessments have been developed is to use risk-based analytical techniques such as the Monte Carlo simulation to incorporate measures of uncertainty in plan evaluation and then to describe the results through the average (expected value) of measures such as National Economic Development (NED) associated with each alternative. Plan selection is then based on the use of these expected values. A better methodology goes beyond expected value by explicitly introducing measures of risk associated with each alternative into the decision process. This approach presents to decision-makers, for example, the option of choosing a plan with more certain, but lower, net NED benefits over a plan with somewhat larger net NED benefits, but a much greater degree of uncertainty as to whether those benefits would actually be realized.

The techniques and procedures to develop better descriptions of the risk characteristics of alternatives already exist. The challenge is to go beyond the use of expected value in terms of both risk communication and risk management. Expected value is only one of the measures to be displayed to decision-makers—other measures that describe risk and uncertainty should also be calculated and displayed. The decision-maker is then provided a richer description of each alternative and can explicitly consider the risk associated with any given choice. In general, this can be considered to be a Multi-Criterion Decision Making (MCDM) problem, in which risk is explicitly characterized and appropriate risk-reward tradeoffs are considered.

This document describes a three-step process for incorporation of risk analysis into the decision making process:

1. Framing of the problem in terms of risk analysis
2. Application of appropriate tools and techniques to obtain quantified results
3. Use of the quantified results in a structured MCDM process

Basic technical concepts of risk are discussed and a variety of methods of quantifying risk and visualizing risk measures are demonstrated. The MCDM approach is discussed. A worked example shows how risk-based information can be incorporated into a MCDMprocess.

# I. INTRODUCTION

The Corps continues to develop techniques and procedures to analyze the risk associated with Civil Works actions. Techniques for developing risk and uncertainty estimates associated with project costs, benefits and impacts, are increasingly being called for in guidance and used in studies. Risk analysis is slowly becoming a more normal way of looking at things within the Corps. To date, however, there has been little incorporation of risk analysis in the actual decision-making or choice of alternatives. Where risk analysis has been used, it is typically used at the technical levels of plan evaluation, to develop average or expected values, for example for net economic development benefits, which are then used in the decision-making effort. This is an important distinction—a particular plan might have a slightly higher average NED benefit than another alternative, but be much riskier, in the sense that the probability of achieving a lower NED benefit is large. This kind of analysis seldom makes its way into the Corps decision-making. This document is informed by a particular viewpoint—that quantification of risk is possible, desirable and necessary—and that it should be explicitly taken into account in the decision-making process when choosing between alternatives.

The purpose of this document is to assist and encourage the development and use of risk-based techniques within the Corps Civil Works decision-making; particularly, go beyond the typical "expected value" approach for choosing between alternatives. The perspective taken is that expected value is only the most basic measure that can be displayed to decision-makers. Other measures that describe risk and uncertainty should also be calculated and displayed properly. This leads to decision-making where the risk, as well as the 'reward,' is explicitly displayed and taken into account in the decision effort. Thus, the process involves:

- Understanding of concepts, methods and techniques of risk analysis
- Ability to communicate risk concepts and measures appropriately
- Ability to utilize risk measures in decision-making

The contents of this document are drawn from the significant body of work and research done within the Corps and elsewhere on issues of risk. It differs from previous work in that the focus of the document is on specific decision-making aspects of risk analysis—that is, at the point at which information is displayed that represents the range of choices and uncertainties associated with those choices. This is in contrast to the currently typical situation in which risk and uncertainty are by and large hidden from the decision-making process.

Many standard works on risk analysis are oriented towards arenas outside the scope of normal Corps activities, such as risks due to low dose exposures of potentially hazardous materials (i.e., radon, chlorinated hydrocarbons in drinking water) or from introduction of new and/or unproven technologies (i.e., nuclear waste disposal, genetically engineered crops). Many of these references devote a good deal of content to questions of determination of dose-response and threshold impacts in humans to biological factors, communication of unfamiliar technologies to the public at large and other matters not highly relevant to the Corps decision issues. In these situations, risk analysis is frequently used to set a target level (e.g., maximum permissible exposure to a pollutant) with no emphasis on how to accomplish that. In the Corps efforts, risk analysis is commonly directed towards evaluating alternative ways of achieving a target.

This document is oriented more towards the typical problems confronted by the Corps analysts and decision-makers, which tend to be related to familiar engineered works, natural processes with historical records and phenomena that are at least generally familiar to the public. Thus, in one sense, the incorporation of risk analysis in decision-making within the Corps is an easier task than in other arenas, because of the long history of analytical efforts at rational decision-making within the Corps and the familiar problems that are examined.

## ROAD MAP

The sections of the guidebook are as follows:

- The Nature of the Problem—Choice Under Risk and Uncertainty in the Corps Civil Works
- Methodologies for Risk Assessment
- Decision-Making Approaches
- An Illustrative Example

Much of the discussion assumes a basic familiarity with risk approaches and measures of risk. An overview of these and related concepts is provided in appendices on:

- Risk in the Corps Planning Process
- Basic Risk Concepts
- The Quantitative Description of Risk
- Risk Visualization and Communication

## ACKNOWLEDGEMENTS

As noted above, this document draws heavily upon the work of others. The project officer is Michael R. Walsh of the Institute for Water Resources. Mr. Walsh has guided development of many of the risk-based and multi-criterion decision tools developed by IWR. Dr. David Moser, (TITLE) of IWR and Dr. Charles Yoe originally created much of the material in this document as part of a Corps training course on Risk Analysis, from which many of the basic concepts are drawn. A critical examination of the Corps approaches to risk analysis, conducted by Prof. Jery Stedinger of Cornell University, was also of great value, as were other previously published Corps documents relating to risk analysis. William Werick of IWR provided information and insight on the use of systems dynamics and shared vision-planning approaches. Keith Hofseth and Bruce Carlson of IWR provided valuable review comments to drafts of this document, as did David Moser and Charles Yoe.

# II. THE NATURE OF THE PROBLEM: CHOICE UNDER RISK AND UNCERTAINTY IN THE CORPS CIVIL WORKS

Within the Civil Works program of the Corps, there is a long history of encouragement, development and use of rational decision-making techniques based on sound, quantifiable engineering, economic, environmental and other factors. The Corps is a major user of cost-benefit analysis as a formal evaluation technique. This requires careful quantification of many factors and numerous techniques have been developed to this end. The Corps guidance dictates choosing, as the preferred alternative, the plan that maximizes net National Economic Development (NED) benefits, subject to certain constraints. The Corps guidance also encourages and frequently requires the use of risk analysis.

The Corps Civil Works Program is primarily concerned with works related to natural processes and events. These processes have inherent natural variability and associated risk and there is uncertainty in our analytical representations (models) due to our inability to completely understand and describe these processes (knowledge uncertainty). Floods, hurricanes and droughts are all beyond point-specific prediction at present. As well, the impacts of natural events have profound economic, environmental and safety implications. Statistical measures can provide important insight into the variability associated with these processes and their consequences. The incorporation of the factors describing uncertainty and variability into the decision-making process comes under the general term of risk analysis, which in itself incorporates the three components of risk assessment, risk communication and risk management. Risk analysis provides a more accurate representation of our knowledge of a particular situation, even if it describes how uncertain our knowledge is. As such, it is an important decision-making aid.

Risk analysis has been used in a number of Corps studies and is applicable to a wide range of decision problems within the Corps. Risk analysis is not only for extreme or low-probability events, but also for any situation in which there is a range of possibilities. At present, there is a body of successful experience within the Corps in the use of risk analysis, including:

- Hoover Dike (SAJ)
  Examination of economic benefits of rehabilitation of the levee surrounding Lake Okeechobee, taking into account uncertainty in hurricanes, lake stage and levee performance.
- Hydropower Rehabilitation Studies
  Comparison of alternative rehabilitation plans for generators/turbines, making use of probabilities of failure of generators and turbines and maintenance and rehabilitation costs.
- Gulf Intracoastal Waterway
  Examination of navigation improvement plans for portions of the GIWW, based on minimizing delays to barge traffic and incorporating uncertainty in tow trips and travel time.

- HEC Flood Damage Assessment
  HEC has developed the widely applied HEC-FDA program, an integrated software system for performing an integrated hydrologic and economic analysis during the formulation and evaluation of flood damage reduction plans. HEC-FDA embodies risk-based analysis procedures to quantify uncertainty in discharge-exceedance probability, stage-discharge and stage-damage functions. HEC-FDA approaches have been used in the Beargrass Creek Basin Study (Louisville District) and numerous studies by the New Orleans, Mobile, Fort Worth, Galveston, Honolulu, Kansas City, Los Angeles, Omaha, Portland, San Francisco, Savannah, St. Louis and St. Paul Districts among others.

The Corps has promoted research into the use of risk analysis approaches, developed specific tools for risk-based analysis in problem arenas important to the Corps and encouraged application of risk analysis as a decision making tool through appropriate guidance. The techniques of risk analysis as applied to the Corps Civil Works problems are reasonably well known and risk analysis applications are increasingly present in the Corps studies. There is increased understanding in the technical community that risk perspectives and approaches provide more useful information about the nature of problems and decisions and this is reflected in requirements for risk analysis in the Corps guidance.

Risk analysis can be viewed as having three components—risk assessment, risk communication and risk management. Risk assessment comprises an analysis of the technical aspects of the problem—what are the uncertainties and what is their magnitude? Risk communication deals with conveying the information, while risk management involves the decision process. Thus, risk assessment tends to provide quantitative (and qualitative) estimates of the results of alternatives containing measures of the risk associated with those alternatives. This aspect of risk analysis has been utilized in a number of Corps projects. Effective communication of the risk-associated results of decision makers and decision-making based on the risk measures has not been realized to the same degree.

In general, the analytical approaches of risk assessment use the tools, techniques and language of probability and statistics to provide estimates of the mean (expected value, average) of parameters related to alternatives in question—average net benefits, average tow delay, average power production. The analytical techniques also provide measures of the risk, that is, the likelihood of achieving the expected result. These risk-related measures add additional variables to the decision process, beyond use of the expected value. Consequently, the decision process becomes more complex. Decision-makers can now include this additional information, which is, in general, less familiar and less easily understood than the more typical measures such as average net benefits. Decision-makers face a difficult task when risk analysis is used:

- Decisions cannot easily be reduced to comparisons of single numbers.
- The decisions are inherently multi-criterion.
- Expert opinion is often involved and expert opinions frequently vary.
- Public perceptions and values may be very distinct from those of decision-makers or analysts.
- Results are less easily interpreted.
- The concepts and techniques are often unfamiliar.

- It is often difficult to communicate effectively about risk and uncertainty issues.
- The ‘best choice’ may not be obvious.

Risk analysis is thus more complex than ‘typical’ benefit-cost analysis:

- It is more explicit about the variables that play a role.
- The variables are generally characterized by more than a single number.
- It is more exact about the state of knowledge of the situation.
- More specificity about interactions between processes is required.
- Some form of computer-based modeling is typically used.
- The models may be complex, hard to understand and evaluate.
- More information is required.

The problem can be addressed by explicitly incorporating descriptions of the variability associated with the different criteria that are relevant to the decision process. This can be done in a variety of ways, from simple descriptions to complete mathematical analyses and definition of probability distributions. The general problem is to go beyond the typical use of measures such as expected value, and incorporate the other, more complete measures of risk. This is not as neat a problem as simply choosing the plan with the largest expected value of net benefits. There are, however, a number of methods that can be used to assist in decisions of this kind. Later sections of this document demonstrate approaches and methods to incorporate this information into the decision process.

## BEYOND EXPECTED VALUE

> “Feasibility studies in water resources are conducted under the assumption that the ‘most likely’ or ‘expected’ values of benefits (monetary and otherwise) can provide an appropriate basis for the evaluation and comparison of alternative plans. Current methodological frameworks are not able to accommodate ranges of values for all input parameters. Even when the effort is made to consider parameter ranges, it is not clear how to proceed with the analysis of risk and uncertainty—what analytical tools are to be used, and what tradeoffs to generate in order to assist in the decision-making process.”
>
> “Risk and uncertainty ought to be addressed more prominently ... Requirements for specification of only expected values should be extended to include a range of possible values and probabilities of occurrence for these values. Again, greater effort should be made to promote the use of risk-assessment techniques in the evaluation of costs and benefits, measurement of effects, and the generation of tradeoffs among alternative plans.” (Goicoechea, 1982)

As typically used within the Corps, risk assessment results in statistical measures related to plan alternatives, usually a mean (expected) value of NED benefits, and a standard deviation. For example, net NED benefits associated with a set of plan alternatives might be as shown in Table II-1.

| TABLE II-1<br>NET NED BENEFITS ASSOCIATED WITH A SET ALTERNATIVES | | |
|---|---|---|
| **Plan Alternative** | **Expected Value of net NED Benefits (million $)** | **Standard Deviation of net NED Benefits (million $)** |
| A | 5 | 4 |
| B | 4 | 2 |
| C | 2.6 | 3 |

Assuming a normal distribution based on the parameters, these options might appear as follows in Figure II-1

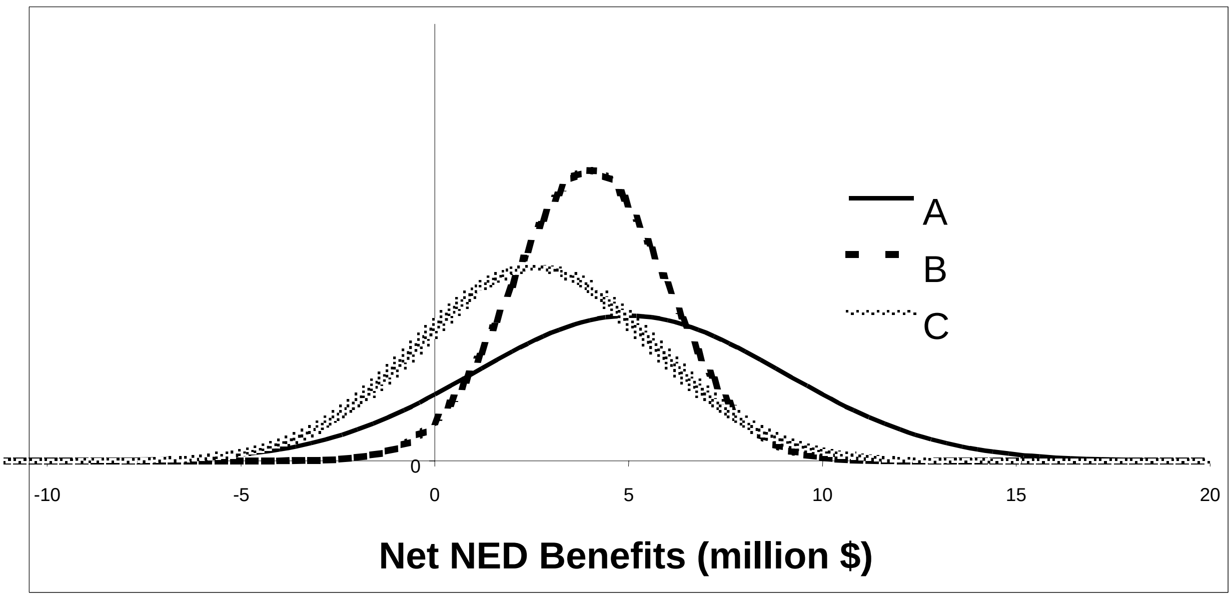



**FIGURE II-1**
**NORMAL DISTRIBUTION OF NET NED BENEFITS**

The location of the peak on the X-axis indicates the mean (expected value). Comparing these values in the chart (and from the table above), Plan A has a higher expected value than Plan B, but the chance of negative net NED benefits is also greater. At the same time, the chance of positive net NED benefits greater than 5 million dollars also greater under Plan A than under Plan B. Under Plan B, the results are less variable. The problem for the decision-maker is to determine whether the greater risk of Plan A is offset by the greater expected value.

Comparing plans B and C, it is clear that most would prefer Plan B to Plan C – the expected value is greater, and the variance is less. There is only a small probability that the net benefits associated with Plan C would be greater than those associated with Plan B, and a much larger probability that the benefits would be lower.

Rather than simply choosing Plan A because of its higher net NED benefits, decision-makers should take into account the additional information that describes the risk associated with each plan. A variety of mechanisms for incorporating such risk information in the decision-making process are described later in this document.

Expected value is frequently ignored in real-world decisions. Gambling and insurance are two examples:

"The mathematical probabilities indicate that we will lose money in both instances. In the case of gambling, it is statistically impossible to expect – though possible to achieve – more than a break-even, because the house edge tilts the odds against us. In the case of insurance, the premiums we pay exceed the statistical odds that our house will burn down or that our jewelry will be stolen.

Why do we enter into these losing propositions? We gamble because we are willing to accept the large probability of a small loss in the hope that the small probability of scoring a large gain will work in our favor; for most people, in any case, gambling is more entertainment than risk. We buy insurance because we cannot afford to take the risk of losing our home to fire – or our life before our time. That is, we prefer a gamble that has 100 percent odds on a small loss (the premium we must pay) but a small chance of a large gain (if catastrophe strikes) to a gamble with a certain small gain (saving the cost of the insurance premium) but with uncertain but potentially ruinous consequences for us or our family" (Bernstein, 1998)

## TYPICAL RISK DECISION-MAKING PROBLEMS IN THE CORPS

While there are many 'flavors' of risk decision-making problems, there are certain types that are common for the Corps decision-makers.

### Single Investment Decision

Problem: Is an investment worthwhile? Which alternative plan is the best? Choose the 'best' alternative, taking into account uncertainties in benefits and costs

Objective: Maximize expected net benefits subject to other constraints

Example: Major Rehabilitation for a Navigation Lock

Uncertainties: Costs of Repair, Structural Integrity of Gates, 'True' B/C ratio

Tools: Monte-Carlo simulation, multiple-criteria decision-making techniques

### Multiple Investment Decision

Problem: Choose a subset of alternatives (more than one) to implement under budget constraints for a given area and objective

Objective: Maximize Expected Total Net Benefits

Example: Allocate money to a number of different wetlands improvement projects

Uncertainties: Costs of improvement, habitat units obtained

Tools: Portfolio Analysis, Incremental Cost Analysis

### Policy/Regulatory

Problem: Articulate a policy to prevent or minimize damage from uncertain future events)

Objective: Hold population exposure to risks to lower than a defined quantity

Example: Dam Safety Policies

Uncertainties: Costs, structural integrity, hydrologic events

Tools: Expert opinion techniques (Delphi, Expert Choice)

### Budgeting

Problem: Allocate resources among a number of competing projects / groups

Objective: Spread funds adequately to support needed activities

Example: Annual O&M Budgeting Process to select work packages

Uncertainties: Costs and outputs of competing work packages, need for emergency funds

Tools: Automated Budget System (ABS), ABS-MCDM (Multi-Criteria Decision-Making)

## A Paradigm for Use of Risk Analysis in Decision-Making

The building blocks of incorporating risk analysis into the decision making process are:

- Framing of the problem in terms of risk analysis
- Application of appropriate tools and techniques to obtain quantified results
- Use of the quantified results in a structured decision-making process

In order to frame the problem appropriately, it is essential to understand risk concepts and have a strong understanding of the problem at hand. This understanding is inherently multi-disciplinary and should be done as a team effort. There are many factors involved – engineering, economic, environmental, etc. In order to capture the diverse fields of knowledge involved, it is useful to develop a document that describes the issue in terms of the:

- Scope of the problem to be addressed
- Engineering and economic methodologies to be applied
- Variables that are taken to be uncertain, and the initially proposed methods of handling risk and uncertainty for those variables
- Sources of data to define the variables

Typically, some type of computerized model is used to develop the quantified results. A variety of such models are used in risk assessment – such as decision trees, spreadsheets, Monte Carlo simulations. For a given problem, there is usually a "build or buy" decision in terms of choosing the appropriate tool. Models can be built 'from scratch' in a spreadsheet or using a programming language. Generic simulation techniques embodied in 'off-the-shelf' programs can be used, or specialized, domain-specific applications, such as those developed by the Corps' Institute for Water Resources for studies in hydropower and navigation can be obtained. All such models need to be examined for at least the following:

- Good congruence with the problem at hand
- Clarity of operation
- Theoretical correctness
- Reasonableness of data requirements
- Adequate documentation of internal mechanisms and procedures
- Ability to trace out what is really happening
- Learning curve
- Ease of use
- Support

If no available model is adequate for the problem at hand, the "build" decision may be exercised. It is essential that any such custom-built effort be appropriately documented, and careful consideration must be given to issues of:

- Resources available for development
- Schedule
- Needed skills to develop, use, and understand the model
- Ongoing support.

Building a tool does not necessarily imply building it in-house – it can be commissioned from a variety of outside sources (e.g., consultants, centers of expertise within the Corps).

Once a model is in hand, it must be used properly. There are many issues involved in insuring that numerical results are reasonable. Monte Carlo simulation is a good example of these issues. Monte Carlo simulation is a well-known technique for analyzing physical systems where probabilistic behavior is important. Events are represented as probabilistic occurrences, with defined probability distributions. The relationships of these events are embodied in the model, which is then run many times, with varying inputs based on the probabilities of the events. The results are recorded for each simulation run (iteration), and summarized statistically. In this fashion, the interacting probabilities result in statistics for the total system. A wide range of situations can be examined along with alternative designs and operations. The result is both an expected value and a distribution of results.

Among the issues that must be addressed for a Monte Carlo simulation are:

- Adequate number of iterations to achieve statistical validity
- Method of data development to parameterize the model
- Appropriate choice of distributions
- Appropriate use of random number generators internal to the model

Similar numerical issues apply to other risk assessment techniques.

The results of a model can provide varying levels of detail for each alternative considered:

- Expected value of parameters
- Other statistical measures of the parameters
- Complete description of the probabilistic distribution of the parameters

As noted previously, it is common for the decision-makers to make use only of the expected value of parameters, and make a choice accordingly. A better choice is to include an explicit description of the risk/uncertainty associated with a criterion. Thus, for example, net NED benefits might be a primary and important criterion, described by expected value, but the expected range of such benefits might also be taken into account,, as could the 'worst-case' level of such benefits.

The general paradigm for use of risk analysis in decision-making proposes that a risk assessment process should make use of analytical tools to provide to the decision-making process measures of both the expected value and the uncertainty associated with that value for each criterion and alternative considered. Quantitative models such as Monte Carlo simulation models can be used to develop a definition of the probability distribution surrounding costs, benefits and outputs. This is the most detailed description of the associated uncertainty, but other, simpler measures, ranging from qualitative (e.g., highly unlikely, likely) to summary quantitative measures (standard deviation, worst-case scenario at a 95 percent probability, etc.), can also be developed. Thus, the output of the risk assessment process for decision-making can be considered to be a matrix describing the situation shown in Table II-2.

**TABLE II-2**
**Risk Assessment Matrix**

| Alternative | Mean NED Benefit | Standard Deviation NED Benefits | Mean Environmental Quality (EQ) Score | Worst Case EQ Score |
|---|---|---|---|---|
| A | 1000 | 52 | 234 | 12 |
| B | 1200 | 250 | 150 | 80 |
| C | 950 | 10 | 200 | 190 |

This type of information can then be moved forward into a multi-criterion decision-making process that allows for explicit examination of risk and risk preferences.

A variety of methods are available for working with MCDM problems. As with the risk assessment techniques, it is important to understand these methods, and their basic assumptions.

In general, the MCDM processes involve the steps of:

- Choosing criteria for the analysis

- Rating all alternatives for each criterion
- Determining which alternatives are dominated, i.e. other alternatives are clearly better on all criteria, and discarding the dominated alternatives
- Applying various methods to score and rank the remaining alternatives, generally based on articulating preferences or weights related to the criteria

Unfortunately, in the general case, there is no guarantee, in an MCDM process, that, after discarding the dominated alternatives, a single alternative will unambiguously be preferred to the others. Accordingly, it is necessary to apply methods of tradeoff analysis to determine preference and ranking among the remaining alternatives. Many computer-based decision tools exist to aid the decision maker in performing this analysis.

In summary, the overall process first starts with an interdisciplinary effort at framing the problem, and documentation of this effort. Based on the documented problem structure, a mathematical model of the problem is created. This model is then used to generate numerical results giving the expected value, and other measures of the parameters of interest. The final decision may require consideration of multiple criteria each of which must be evaluated for risk.

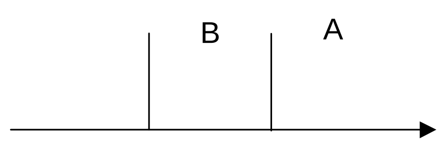
## CHOICE UNDER RISK AND UNCERTAINTY

The nature of the choice problem, when risk measures are used, is illustrated by the following situations.

**Typical – Assumed Certainty**

Plan A: NED Benefits of 1.05 million
Plan B: NED Benefits of 1.03 million

Choose A

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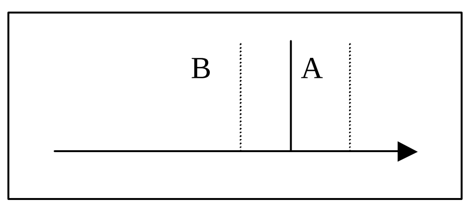
**Under Uncertainty: Guaranteed Amount vs. Range**

Plan A: guaranteed Net NED Benefits of 1.05 million

Plan B: 50 percent chance of Net NED Benefits of .95 million, 50 percent chance of 1.15 million (= expected value of 1.05 million)




There is no clear choice. The decision is made based on risk preferences – is a certain value of 1.05 worth more than an equal chance at .95 or 1.15. Is the gamble worthwhile? A risk-averse decision-maker would prefer the certain 1.05 million (preferring to avoid the possible lower value of .95), while a risk-seeking decision-maker might favor the opportunity to achieve the higher 1.15 value.

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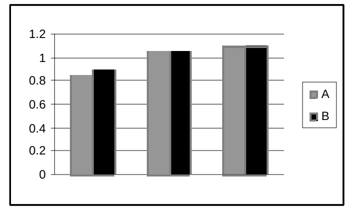
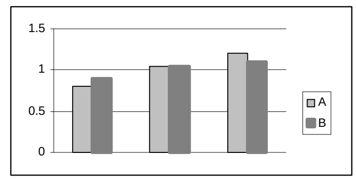
## Stochastic Dominance




Plan A: average Net NED Benefits of 1.05 million, minimum of .85, maximum of 1.10
Plan B: average Net NED Benefits of 1.05 million, minimum of .9, maximum of 1.10

B should be preferred (there is a higher probable minimum, other factors are equal, so there is less exposure to a less favorable consequence.) B is said to be ***stochastically dominant*** over A, because it is at least as good on all possible conditions, and better on at least one condition (higher minimum).

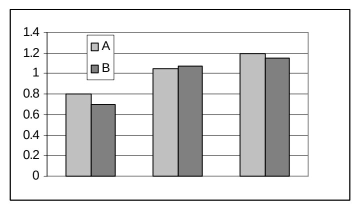
## Preference for smaller range (less variability)?




Plan A: average Net NED Benefits of 1.05 million, minimum probable of .8, maximum probable of 1.20
Plan B: average Net NED Benefits of 1.05 million, minimum probable of .9, maximum probable of 1.10

B may be preferred. There is less uncertainty – the range is smaller, so there is less exposure. There is, however, a larger possible maximum with A. A risk-averse decision-maker might prefer Plan B.

## Slightly higher expected value with increased risk of failure




Plan A: average Net NED Benefits of 1.05 million, minimum probable of .8, maximum probable of 1.20
Plan B: average Net NED Benefits of 1.07 million, minimum probable of .7, maximum probable of 1.15

There is no clear choice. Plan B has a higher expected value, but both the minimum and maximum associated with Plan A are greater. This is a tradeoff situation.

**The addition of risk measures complicates the decision process, but better expresses the state of knowledge about the situation.**

# III. RISK IN THE CORPS PLANNING PROCESS

The Corps guidance specifies that risk and uncertainty will be incorporated in the water resource planning process:

> "The Principals and Guidelines state that planners shall characterize, to the extent possible, the different degrees of risk and uncertainty inherent in water resources planning and to describe them clearly so decisions can be based on the best available information.
>
> Risk-based analysis is defined as an approach to evaluation and decision making that explicitly, and to the extent practical, analytically incorporates considerations of risk and uncertainty.
>
> Risk-based analysis shall be used to compare plans in terms of the likelihood and variability of their physical performance, economic success and residual risks.
>
> A risk-based approach to water resources planning captures and quantifies the extent of risk and uncertainty in the various planning and design components of an investment project. The total effect of risk and uncertainty on the project's design and viability can be examined and conscious decisions made reflecting an explicit trade-off between risk and costs."
>
> (U.S. Army Corps of Engineers Planning Guidance Notebook, ER 1105-2-100, Chapter 2, April 2000)

The Corps planning process is comprised of the following steps:

- Identifying Problems and Opportunities
- Inventorying and Forecasting Conditions
- Formulating Alternative Plans
- Evaluating Alternative Plans
- Comparing Alternative Plans
- Selecting a Plan

Uncertainty is clearly present in forecasts, which carries through all subsequent steps. The recognition of uncertainty in estimates of costs, benefits, and impacts falls under the evaluation step, and is the arena in which risk assessment is most prevalent within the Corps. Comparison of alternative plans has generally been done based on expected value, without extensive display of the risk-related issues. The selected plan is frequently (but not always) the plan with the highest expected value.

Opportunities exist to enhance the communication and display of risk characteristics in plan comparison, and to use improved decision analysis techniques taking into account risk in plan selection.

The work of the Corps is organized within nine business processes:

- Navigation
- Flood Control / Coastal Protection
- Environmental Impact / Ecosystem Restoration

- Hydropower
- Water Supply
- Recreation
- Emergency Management
- Regulatory Functions
- Support For Others

Risk and uncertainty play roles in many of these arenas, with varying levels of application of tools, techniques, and models to date. Some example applications and guidance are given in the accompanying Appendix B on "Risk In the Corps Planning Process."

# IV. METHODOLOGIES FOR RISK ASSESSMENT

Methods for performing risk assessments are well documented elsewhere (U.S. Army Corps of Engineers, IWR, 1992). The basic approach involves:

- Identification of the important processes and variables (input and output)
- Determination of the arenas of uncertainty, and parameterization of the uncertain variables
- Development and/or use of techniques (typically modeling) to combine the input uncertainties to determine the distribution of outputs

A number of tools and techniques have been developed to allow for characterization, computation, and / or analysis, including:

- Influence diagrams
- Decision trees
- Systems dynamics models
- Monte Carlo simulation

Many of these techniques are embedded in software for model construction that allows for graphical model creation and parameterization, and then provides numerical solutions.

## Influence Diagrams

Influence diagrams are an excellent method for organizing and displaying the inter-relationships of complex processes and models. They can graphically display the modeler's view of the system, incorporating displays of the role of uncertainty.

"In many cases the mathematical model, often expressed as a series of equations, serves poorly as a tool for communicating or structuring a model. A graphic device called an influence diagram will both display the problem and frame the concept of the model. Displaying it clearly shows the chief beliefs embodied in the model. Such a presentation can assure a user of the model or a client of the model builder that a messy situation has been understood and brought under some control. The influence diagram is a display of all of the decisions, variables, and outcome attributes that pertain to a problem, along with the influence among them." (Bodily, 1985.

A variety of drawing conventions can be used for influence diagrams, but, in general, different shapes are used to represent different kinds of information such as uncertain values, decision variables and intermediate variables. These shapes are connected by arrows showing how each variable influences other variables. A variety of software packages are available for construction of influence diagrams, including Analytica (Lumina Decision Systems, http://www.lumina.com/) and Precision Tree® (Palisade Software, http://www.palisade.com/html/ptree.html).

A simplified example of such a diagram, relating to the economic analysis model used for determining damages due to levee breaching at Hoover Dike surrounding Lake Okeechobee, is shown in Figure IV-1.

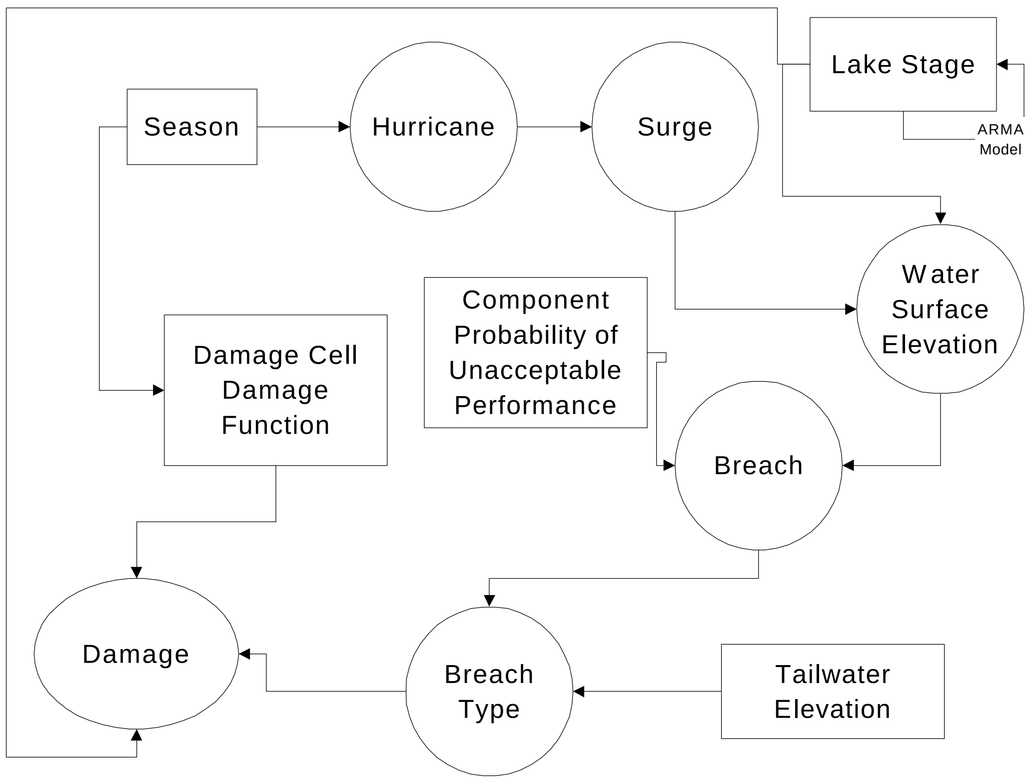



**FIGURE IV-1**
**VARIABLE INFLUENCE DIAGRAM**

## Decision Trees

Decision trees are another form of graphical display of problem structuring, particularly well suited for limited situations where the possible events can be well defined, such as in major rehabilitation analyses. Decision trees are more explicit definitions of the options associated with various paths than in the influence diagram and are represented as a branched node-link network. The decision tree shown in Figure IV-2 represents a simple analysis of the advisability of installing a flood warning system, at a cost of $20000. If the probability of flooding is .1, and the damage associated with flooding in the absence of the warning system is $50000, but is reduced to $5000 with the flood warning system, then the decision tree analysis suggests that the preferred choice is not to build the flood warning system. While this simple decision tree is based on expected value analysis, more complex decision tree technologies do provide more complete risk profiles.

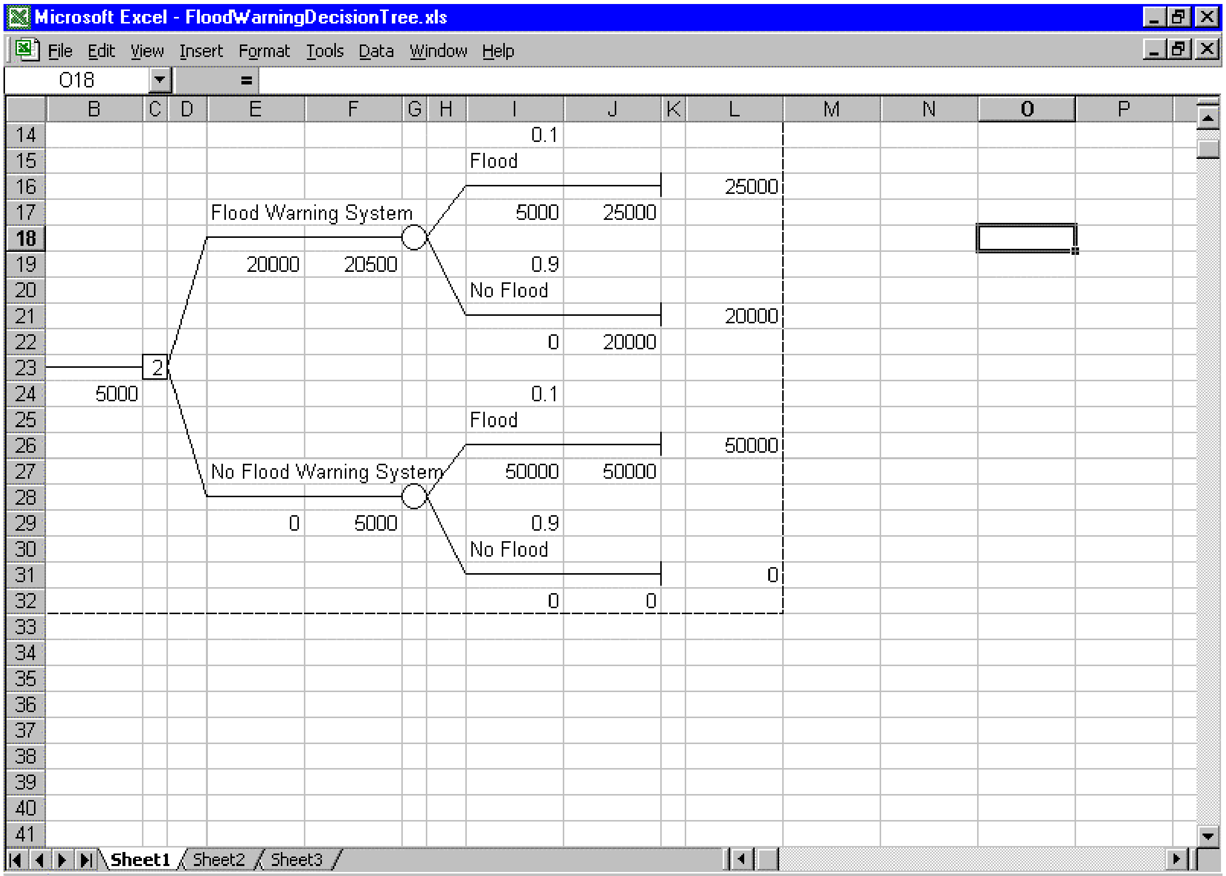



**FIGURE IV-2**
**DECISION TREE**

Nodes can represent events or decisions. Event nodes usually have multiple possible branches, with associated probabilities. The initial decision, at the left-most node, is whether or not to deploy a flood warning system. Subsequent nodes at the next level to the right represent possible states, and the associated probabilities, in this case the probability of a flood. The tree is both a form of display and a form of calculation. When consequences are assigned to the ultimate leaves of the tree, the tree can be "rolled back" to determine, using the rules of probability and decision rules at decision nodes, the 'best' choice (i.e. the decision that maximizes or minimizes the desired value) based on the various event probabilities. More sophisticated decision tree software allows for more complex specification of probability distributions, and, through use of Monte Carlo simulation techniques, can display probability distributions for desired decision variables.

"The influence diagram and decision tree show different kinds of information. The influence diagram shows the dependencies among the variables more clearly than the decision tree. The decision tree shows more details of possible paths or scenarios as sequences of branches from left to right. But, this detail comes at a steep price: First, you must treat all variables as discrete (a small number of alternatives) even if they are actually continuous. Second, the number of nodes in a decision tree increases exponentially with the number of decision and chance variables. ...The influence diagram is a much more compact representation." ( Lumina Decision Systems, http://www.lumina.com/)

## System Dynamics Models

System dynamics (SD) models represent a particular problem (e.g., water supply planning) in a time-based simulation framework, composed of standard building blocks (‘stocks’, ‘flows’) that can be parameterized to suit a particular situation. The model-building tools are general in nature, and can be used for a variety of different applications, through construction of the appropriate model. System dynamics models have been used successfully within the Corps as part of the shared vision modeling effort in which stakeholders participate in the construction of simulation models. Graphical model building and parameterization, and ease of construction and use are important features of SD models. The Stella (tm) software package has been used in a number of projects in the Corps, including:

- Cedar-Green River Drought Preparedness Study, Seattle-Tacoma Washington, 1990-93
- New River Drought Preparedness Study, West Virginia, 1990-93
- James River Drought Preparedness Study, Virginia, 1990-93
- Boston Area Drought Preparedness Study, Boston, MA, 1990-93
- Marais de Cygnes-Osage Rivers Drought Preparedness Study, Kansas and Missouri, 1990-95
- Comprehensive Study of the Alabama-Coosa-Tallapoosa-Apalachicola-Chattahoochee-Flint 1994-1996
- Devils Lake Decision Support Study, 1997-98

System dynamics modeling allows for the representation of uncertainty by assigning distributions to variables. The major strength of SD modeling is the ease of defining and visualizing the interactions between the components of a system. STELLA is a simulation model and can support some, but not all the mathematical calculations that inform decisions where risk and uncertainty are involved. It is weakest in performing repetitive calculations such as would be used in numerical analysis or Monte Carlo simulations. It is usually a good or very good tool for performing sensitivity analysis, and this is the mechanism by which most uncertainty evaluations have been carried out to date in the Corps efforts.

The strength of shared vision planning is that it develops trust in the basic analytic package, and encourages stakeholders to develop quantified performance measures for each planning objective. This increases the chances that risk assessments can be done and will be relevant to decision makers and stakeholders. As an example, in the Marais de Cygnes-Osage Rivers Drought Preparedness Study, Kansas and Missouri, the proposed drought plan met everyone’s expectation when tested with a recurrence of historic 1950s drought, but there were questions about performance in a more severe drought. By creating a synthetic six year stream flow based on a triple repeat of the worst two year flows and correlating those flows to a six year precipitation volume that was estimated (using the National Drought Atlas) at a 1000 year recurrence interval, a worst case scenario was created and evaluated. All parties agreed the hydrologic method was reasonable, if arguable, and that the resulting flows were very low. The drought plan barely satisfied needs during this hypothetical drought, but that was reassuring to decision makers - they had at least considered the worst-case scenario mathematically and rationally.

In the Comprehensive Study of the Alabama-Coosa-Tallapoosa-Apalachicola-Chattahoochee-Flint, additional uncertainty evaluation was carried out by using sensitivity analysis, including impact of uncertainty of forecasts of agricultural production, barge traffic, levels of water conservation, and groundwater pumping.

The diagrams in Figure IV-3 and IV-4 show screen captures from the Corps Devils Lake Stella model, showing the internal model structure and a simple user interface screen.

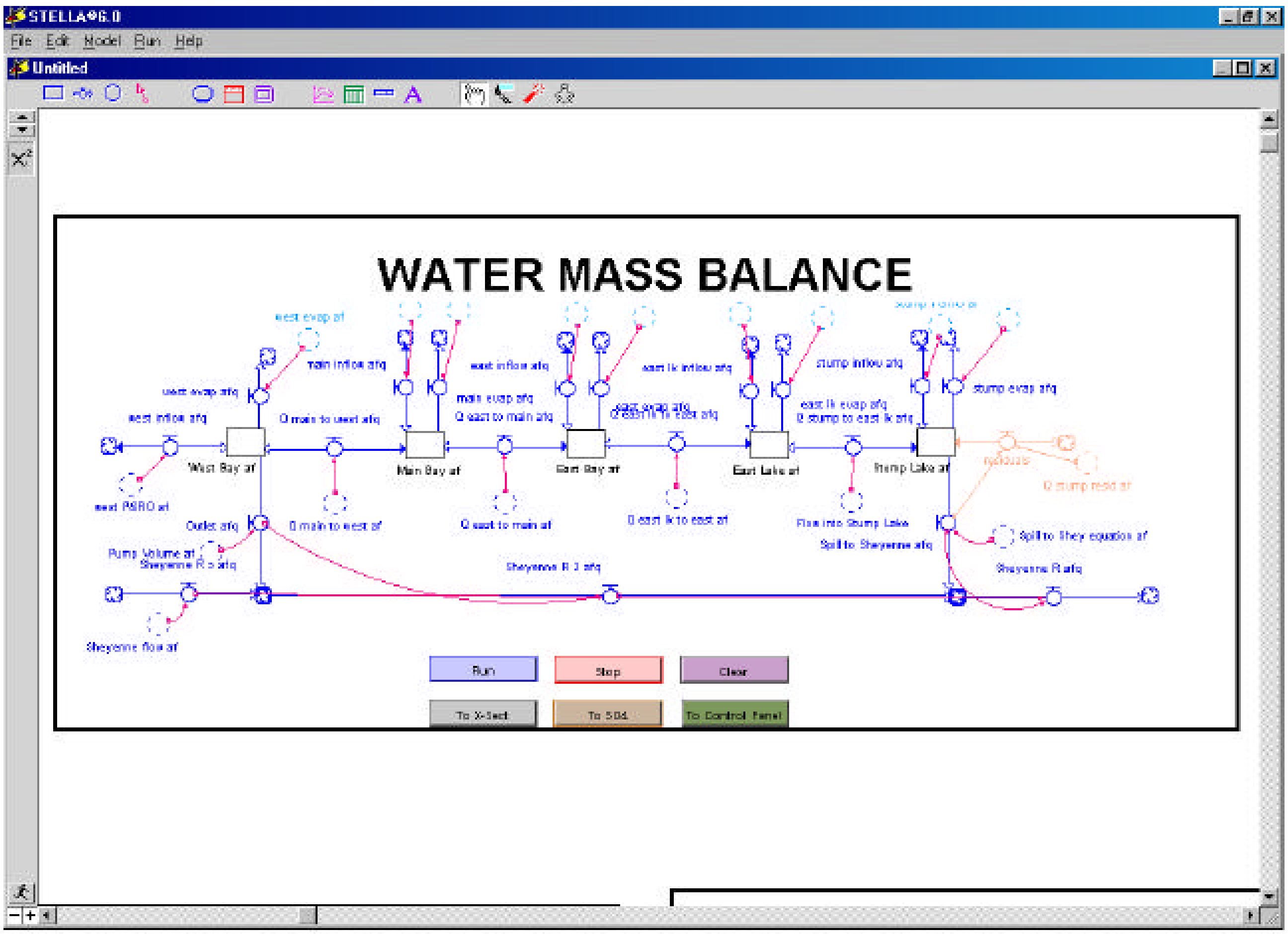



**FIGURE IV-3**
**INTERNAL MODEL STRUCTURE (DEVIL LAKE STELLA MODEL)**

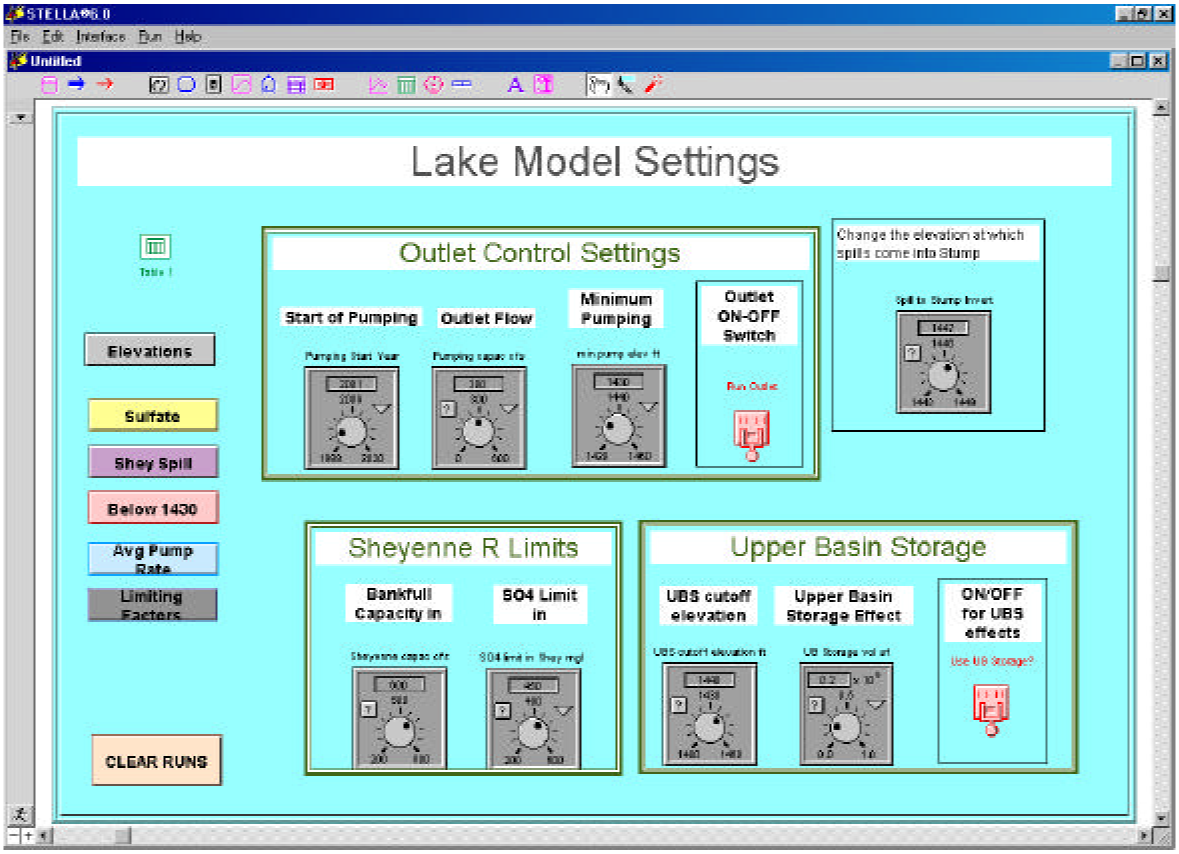



**FIGURE IV-4**
**USER INTERFACE SCREEN CAPTURE (DEVIL LAKE STELLA MODEL)**

## Monte Carlo Simulation

The complexities of the combined engineering-economic problem of risk analysis, in which there are uncertainties associated with the physical performance of systems, and the economic consequences of that performance, are typically addressed through the use of *Monte Carlo simulation techniques*. The complexity of the underlying problem usually dictates that there is no closed-form, analytical solution. Instead, computer-based simulation is used to provide numerical characterization of the behavior of various alternatives. Monte Carlo simulation is particularly useful for physically based real-world problems, where the results of the simulation can be tested against historical and reasonable behaviors.

Monte Carlo simulation combines uncertainties in many variables that describe a system, to obtain a statistical description of the behavior of the system as a whole. This is accomplished through repeated runs of a simulation model, varying the input data based on the statistical descriptions of uncertain parameters. Monte Carlo techniques can be used with existing deterministic models, by varying the input parameters and repeatedly running the model. Such an approach was used by the Jacksonville District in adding uncertainty to an existing storm damage model used for shore protection studies.

The important sources of uncertainty for a given problem (failure rates, for example) are identified, and described statistically. These data are then used within a simulation model developed for the problem. At each point within the simulation at which descriptive data for uncertain variables are required, the Monte Carlo distribution is used to select a value (through random sampling from the distribution), which will thus differ from simulation run to simulation run. Many iterations are calculated, and the resultant overall statistics are used in the decision-making process. A large amount of data must be managed, and the simulation must provide sufficient information to allow for validation and verification.

Critical issues for effective use of Monte Carlo simulation are: appropriate abstraction and definition of the problem; efficiency of computation, to allow for multiple iterations and associated statistical validity; management of input data and ease of use; and analysis, verification, and visualization of results.

Monte Carlo simulation is probably the most widely used risk assessment technology within the Corps. Spreadsheet-based techniques (such as @Risk ®, http://www.palisade.com and Crystal Ball®, http://www.decisioneering.com) custom programs, and generic tools (such as the suite of problem-specific tools available from IWR) have all been used to analyze a variety of problems, including major rehabilitation for hydropower, shore protection, and navigation improvement studies.

Since Monte Carlo is an approach to risk/uncertainty modeling, rather than a specific model, the technique can be added to other, deterministic models, such as decision trees. System dynamics models can be run repeatedly to obtain a distribution of outputs. The important features of the Monte Carlo simulation approach are:

- Assignment of probability distributions to input parameters
- Repeated runs of the model (iterations), varying the inputs based on the probability distributions
- Use of information generated by each iteration to develop the statistical distribution of output parameters

## Example Outputs of the Risk Assessment Process

Traditionally, the risk assessment process within the Corps has used models to provide the mean, or expected value, of parameters associated with each alternative. As noted previously, this is not sufficient. At minimum, some measure of the risk associated with achieving the mean value should be displayed, typically in the form of a range, or standard deviation. At the maximum, the entire probability distribution associated with the parameters can be displayed. The following shows a portion of the output associated with the Hoover Dike Model, showing information on cost statistics:

| *Scenario Description Summary* | | | *15-Oct-98* | *5:17 PM* |
|---|---|---|---|---|

| | | | |
|---|---|---|---|
| **Scenario:**T-1000 | **Description:** 1000 Iterations | | **Run Date:** 8/21/98 2:44:25 PM |
| **Plan:**None | **General Lake Statistics** | | **Simulation Settings** |
| | **Initial Lake Level Mean:**15 | **Critical Tail Water:** 13 | **Iterations:** 10 |
| **Start Year:** 1997 | **Initial Lake Level Std Dev:**1 | **Max Lake Level (ft):** 24 | **Cycles:** 200 |
| **Base Year:** 2000 | **Static Lake Level Mean**14.379 | **Min Lake Level (ft):** 10 | **Interest Rate:**0.07125 |
| **Hurricanes:** Yes | **Static Lake Correlation:**0.535 | **Lake Moving Average:** -0.4856 | **Seed:** 0 |
| **O & M Multiplier:**1 | **Static Lake Std Dev:**1.5153 | | |

| | **Average Cost ($)** | **Min Cost ($)** | **Max Cost ($)** | **Std Dev** | **Std Error** | **Rehab Cost ($):** 0 |
|---|---|---|---|---|---|---|
| **Repair:** | 839,851 | 0 | 2,795,496 | 922,155 | 291,610.9 | |
| **O & M:** | 34,949,223 | 33,803,296 | 36,259,059 | 771,873 | 244,087.7 | |
| **Damage:** | 126,273,058 | 0 | 302,220,238 | 106,764,933 | 33,762,036.4 | |
| **Total:** | 162,062,132 | 106,771,706 | 33,764,178. | 0 | | |

Additional information showing the uncertainty in other parameters is also available:

Random Lake Stage Statistics
Observations: 1000000 Mean: 0.0014 SD: 0.6382 Max: 3.0332 Min: -3.1527

Hurricane Statistics
Code: 0 None Avg: 191.4284 SD: 2.46578 Max: 197.0000 Min: 182.0000

Code: 1 Atlantic Avg: 5.1018 SD: 2.04065 Max: 10.0000 Min: 0.0000

Code: 2 Gulf Avg: 3.4698 SD: 1.97552 Max: 11.0000 Min: 0.0000

Lake Stage Statistics

Annual

Annual: Avg: 14.37826 SD: 1.49826 Max: 21.63107 Min: 10.00000

Period: 1 Avg: 14.38384 SD: 1.49631 Max: 20.84259 Min: 10.00000

Period: 2 Avg: 14.37886 SD: 1.49732 Max: 21.63107 Min: 10.00000

Period: 3 Avg: 14.37386 SD: 1.49876 Max: 21.29789 Min: 10.00000

Period: 4 Avg: 14.37647 SD: 1.50063 Max: 20.94368 Min: 10.00000

Annual Histogram

Number Of Observations: 1000000

Stage Range Percent #

```
[ 9.00 - 10.00] 0.0 0
[ 10.00 - 11.00] 1.3 12569
[ 11.00 - 12.00] 4.5 44714
[ 12.00 - 13.00] 12.3 122816
[ 13.00 - 14.00] 22.0 219552
[ 14.00 - 15.00] 26.0 260017
[ 15.00 - 16.00] 19.9 199466
[ 16.00 - 17.00] 10.1 101251
[ 17.00 - 18.00] 3.2 32441
[ 18.00 - 19.00] 0.6 6443
[ 19.00 - 20.00] 0.1 681
[ 20.00 - 21.00] 0.0 46
[ 21.00 - 22.00] 0.0 4
[ 22.00 - 23.00] 0.0 0
[ 23.00 - 24.00] 0.0 0
[ 24.00 - 25.00] 0.0 0
```

Figure IV-5 is derived from output of the Hoover Dike model, and shows the running average and individual iteration values for a portion of a 5000-iteration Monte Carlo simulation, clearly showing the wide variability among individual iterations.

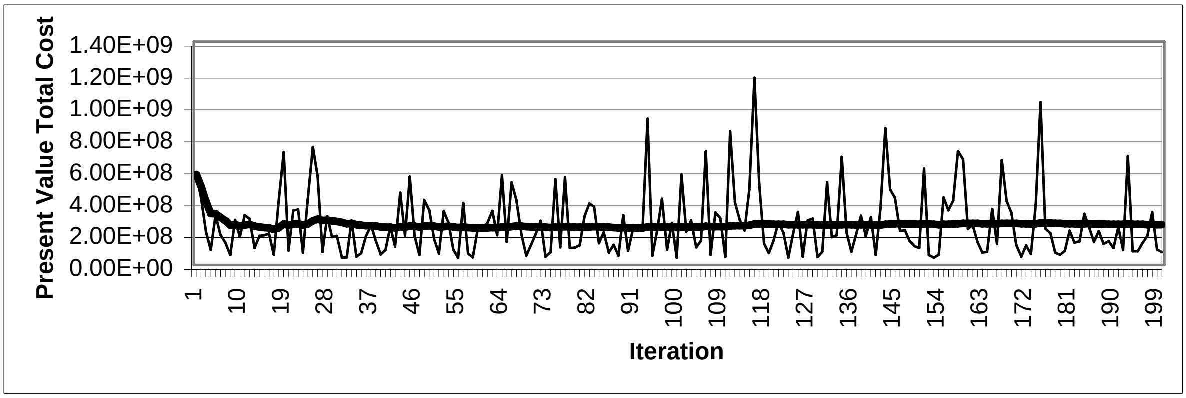



**FIGURE IV-5**
**TOTAL COST BY ITEREATION (HOOVER DIKE MODEL)**

Figure IV-6 and IV-7 are a histogram and cumulative distribution of the total cost associated with this 5000 iteration run.

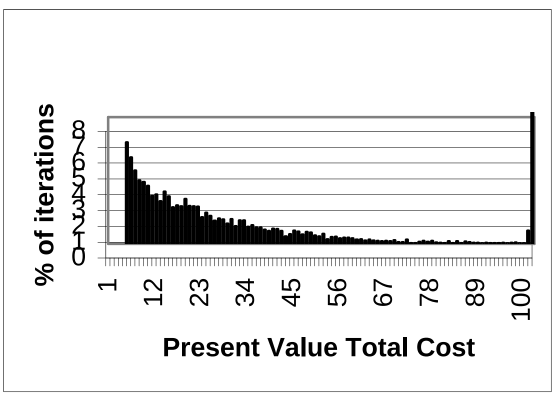



**FIGURE IV-6**
**HISTOGRAM**
**(HOOVER DIKE MODEL)**

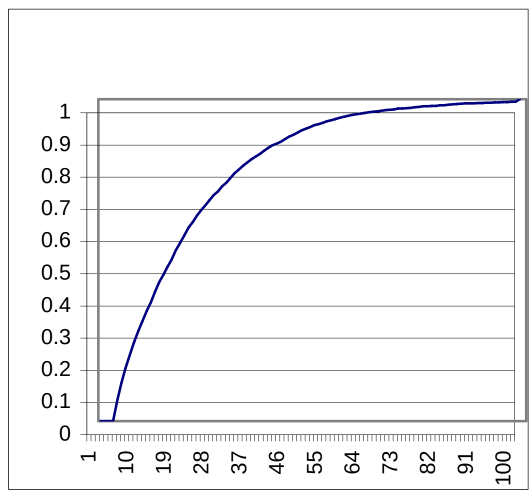



**FIGURE IV-7**
**CUMMULATIVE DISTRIBUTION**
**(HOOVER DIKE MODEL)**

The NavSym model simulates movement of tows on a waterway and, as noted previously, has been applied to studies on the Gulf Intracoastal Waterway. A simulation run moves tows through the waterway from origin to destination,and counts the number of tows, the total time tows spend in the system during the simulation duration and the associated cost. A portion of the output of the NavSym model, showing average, standard deviation, maximum and minimums is shown below.

***Simulation Results***

| Average Tows | SD Tows | Minimum Tows | Maximum Tows |
|---|---|---|---|
| 1,724.367 | 47.458 | 1,625.000 | 1,834.000 |
| Average Hours | SD Hours | Minimum Hours | Maximum Hours |
| 21,494.351 | 854.733 | 19,226.624 | 23,972.924 |
| Average Cost | SD Cost | Minimum Cost | Maximum Cost |
| $3,232,033.01 | $126,131.39 | $2,900,535.38 | $3,592,794.55 |

# V. Decision-Making Approaches

Most Corps decision-making problems are multi-criterion: a number of dimensions of the problem need to be considered: cost, benefit, environmental impact, safety, risk. These dimensions cannot and should not always be reduced to a single measure such as dollars.

Historically, the Corps has used a single measure (net NED Benefits) as the primary method of selecting amongst acceptable alternatives. [More recently, additional criteria such as environmental restoration are being considered in a multiple criterion approach]. The NED determination generally requires a complex analysis to determine benefits in dollars, costs in dollars, and other non-dollar measures (environmental quality, public acceptance, etc.) that determine whether or not a particular alternative is acceptable. The alternative with the highest net NED benefit is the one usually chosen. This is essentially a single-criterion decision process. While the process is not always simple, it is well understood within the Corps.

Traditional Benefit-Cost Analysis does not capture our real state of knowledge about a problem. We know that we don't precisely know the costs, benefits, impacts, and interactions. This realization has been incorporated in many newer risk-based analysis techniques encouraged and promoted by the Corps. For the most part, these techniques are used at a lower level of analysis, such that the uncertainties are used to develop an expected value of outcome (e.g., net NED benefits). When risk and uncertainty are taken into account, it is usually through use of some measure of expected value of benefits and costs associated with an alternative, calculated during the analysis of alternatives. Risk and uncertainty measures associated with an alternative are seldom displayed directly to decision makers. Rather, the expected value of net NED benefits is displayed as the value associated with an alternative, and compared with similarly calculated values for other alternatives. The decision among alternatives is then based upon the choice of expected values. As has been demonstrated, that approach is limiting, and may not always lead to the preferred decision. Using risk analysis techniques, a distribution of outcomes for each alternative can be generated, rather than a single number. The approach recommended in this document is that the uncertainty values for alternatives be carried forward explicitly into the decision making process, along side the expected value.

If risk is explicitly taken into account, added dimensions describing the uncertainty must be included. The uncertainty measures can be incorporated within each criterion, or separated out as separate criteria. Certain decision-making tools and techniques can make use of mathematical distributions associated with criteria, allowing the uncertainty measures to be handled directly, while others require that the uncertainty measures be considered separately, as distinct criteria. The former approach is preferred. At minimum, the expected value of net NED benefits, plus the uncertainty in that value, is measured as a criterion for all alternatives. As shown in the examples above, there is not always a clear choice between two alternatives when both an expected value and uncertainty are displayed.

While there is no general, explicit solution to what is 'best' amongst a group of alternatives, there are a number of well-defined procedures that can be used to assist decision-makers in making choices. The field is generally referred to as multi-criteria decision-making (MCDM) or multi-criteria decision-aiding (MCDA). Aspects of the problem have been studied formally since the 1700's. With the advent of computers and graphical interfaces, a number of interactive computer-assisted decision support systems incorporating MCDA have been developed.

The overall process is generally carried out in phases:

- Information gathering – survey of criteria and possible alternatives
- Evaluation – analyze the alternatives and evaluate them for each of the chosen criteria
- Choice – apply a specific MCDA technique to select among alternatives

This is consistent with the basic steps of the general Corps planning process: problem identification; plan formulation; assessment; and plan selection. Thus, MCDA approaches fit quite naturally into the Corps process.

A more detailed discussion of the general problem of tradeoff analysis is available in Yoe (2002).

## MULTIPLE CRITERION DECISION-MAKING: BASIC CONCEPTS

The following general model of multi-criterion decision making is useful in providing a common terminology and framework for the problem. Other terms are often used in the literature, but the general approach is fairly similar across most discussions of the problem.

A Decision Maker (DM) is an individual or group faced with making a choice among several possible alternatives. Decision makers are assumed to be rational – that is, they do not make inconsistent choices (e.g., preferring Plan A to Plan B, and Plan B to Plan C, but preferring Plan C to Plan A).

**Alternatives** are **discrete** and **distinct** options or plans for the problem under study, e.g.,:

- Dredge channel to 14'
- Dredge channel to 20'
- Provide new moorings

Note that the examination of **continuous** alternatives, with an infinite number of possible solutions (e.g., Dredge Channel to 14.335' feet) is a different type of problem, not considered here.

**Criteria** or **attributes** are (multiple) dimensions on which an alternative is measured, e.g., cost, benefit, environmental impact, etc. One of the roles of the decision maker is that of determining the criteria to be used.

The **analyst** models the situation under study, may frame the alternatives considered and assist in definition of criteria, and provides measures for the criteria for each alternative. The analyst also uses tools to assist decision makers in the process of making choices.

The problem is thus framed as that of choosing among alternatives that are characterized by measures on multiple criteria, where at least one of the criteria is some measure of the risk associated with the alternative. Generically, a ***decision matrix*** is constructed by the evaluation step, and is processed by MCDM techniques. The decision matrix lists all alternatives, criteria and the measures of the criteria shown in Table V-1.

**TABLE V-1**
**DECISION MATRIX**

| Plan Alternative | Expected Value of net NED Benefits (million $) | Standard Deviation of net NED Benefits (million $) |
|---|---|---|
| A | 5 | 4 |
| B | 4 | 2 |
| C | 2.6 | 3 |

The problem is then to make choices based on the decision matrix. A number of different types of choices can be made:

- Choose a single ‘best’ alternative
- Rank all alternatives in order of preference (ordinal ranking)
- Rank all alternatives by strength of preference (scoring, cardinal ranking)
- Group alternatives (e.g., optimal, preferred, satisfactory, unsatisfactory) for further examination or dismissal from consideration

These are not the same problem, and have different approaches, additional data requirements, and philosophical underpinnings. In particular, it is typically necessary for the DM to express some value of preference among criteria, declaring some criteria to be more significant to the final choice than others. Much effort has gone into development of techniques to obtain these preferences from decision makers, with no obvious generally applicable or preferred method.

## Steps in the Process

It is assumed that alternatives have been defined, and criteria established. The typical role of the analyst is then to rate each criterion to create the decision matrix. In some cases, it is possible to eliminate alternatives early (in a process called screening), if they fail to meet some particular desired level on important criteria, reducing the need for detailed analysis.

Once the decision matrix has been generated, *non-efficient* alternatives (see below) should be eliminated. In general, this will not lead to a single alternative that is clearly better than all others on all criteria. This leaves a set of alternatives that must be evaluated using some form of tradeoff analysis. At this point, there is no clear-cut choice between alternatives, without adding some additional information relating to importance of different criteria.

Two methods of further reducing the set of alternatives are elimination methods and explicit tradeoff methods. In elimination or non-tradeoff methods, minimum (or maximum) levels (constraints) on criteria are set, and alternatives that do not meet those constraints are discarded. In explicit tradeoff methods, some relative value is set between criteria, for example a valuation of environmental quality vs. economic benefits. This is often accomplished by assigning weighting factors to the individual criteria. Typically, some form of composite score is then generated for each alternative, and used for ranking of alternatives. The steps in this process are outlined in Figure V-1.

Neither of these approaches is without both theoretical and practical problems. There is no single methodology that can unambiguously choose between non-dominated alternatives without requiring clear preference statements. It is for this reason that many experts in the field prefer the term Multi-Criteria Decision Aid to Multi-Criteria Decision Making, in order to emphasize that the available tools and techniques are designed to display to decision-makers the combined consequences of technical analysis of the alternatives and the preference structure of decision makers.

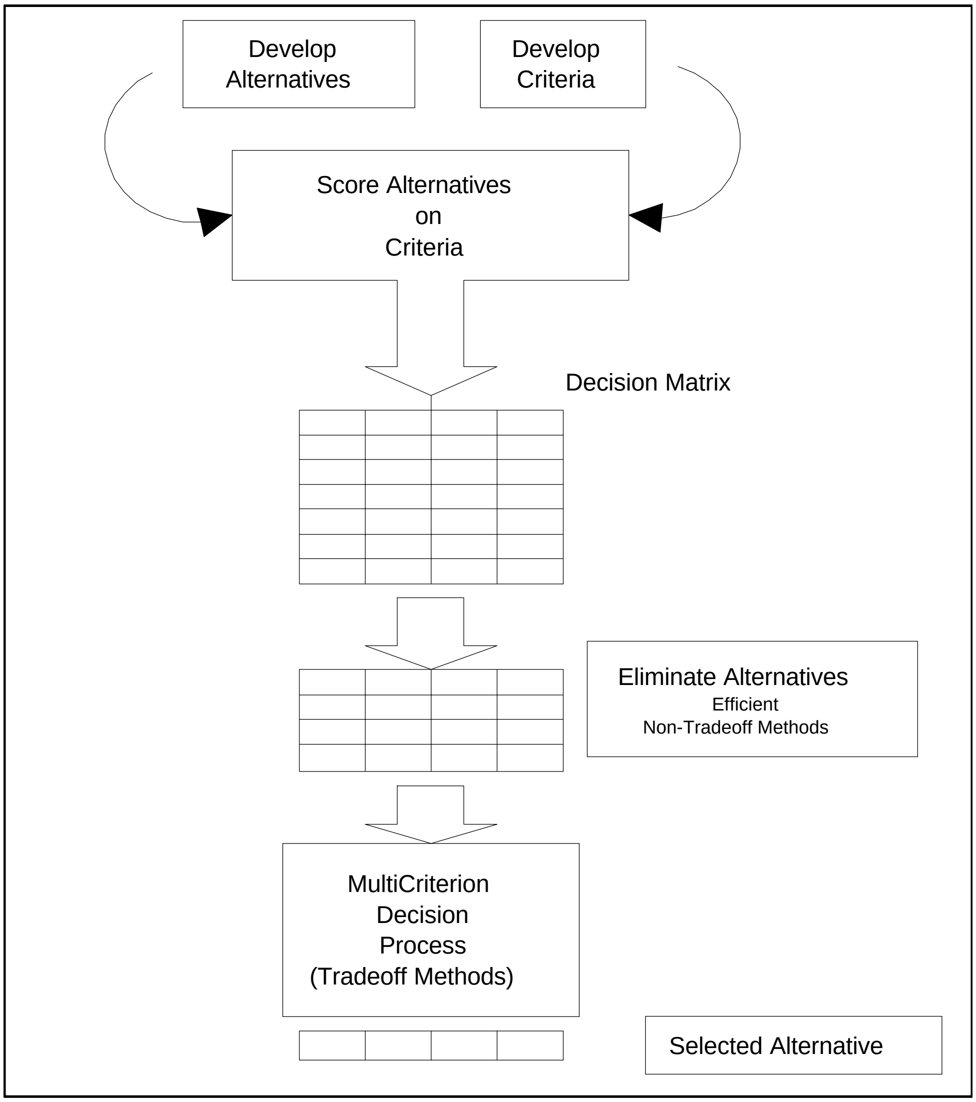



**FIGURE V-1**
**STEPS IN THE MULTICRITERION DECISION PROCESS**

## LIMITING THE RANGE OF ALTERNATIVES – DOMINATION

The first step in MCDM (assuming rational decision-makers) is to eliminate from the choice set any ***inferior*** alternatives (inferior is used as a technical term, not a qualitative judgment), that is, those that are not as good as others in the choice set. Inferior alternatives give less benefit (on all measures of benefit) at greater cost (on all measures of cost) than other available alternatives. An example of this behavior is provided in Table V-2.

**TABLE V-2**
**Dominating Alternatives Example**

| Alternative | Cost Measure | Environmental Quality Measure |
|---|---|---|
| A | 100 | 30 |
| B | 200 | 25 |
| C | 150 | 40 |
| A **dominates** B – it costs less, but gives greater environmental quality. A does not dominate C, because C has greater EQ than A, but also costs more. | | |

Given a number of alternatives measured on different (non-commensurate) criteria:

The non-dominated (efficient) set of alternatives consists of those alternatives where no other alternative is 'better' on all the measures. These alternatives are referred to as the **Efficient** set. A visualization of the efficient set is provided as Figure V-2.

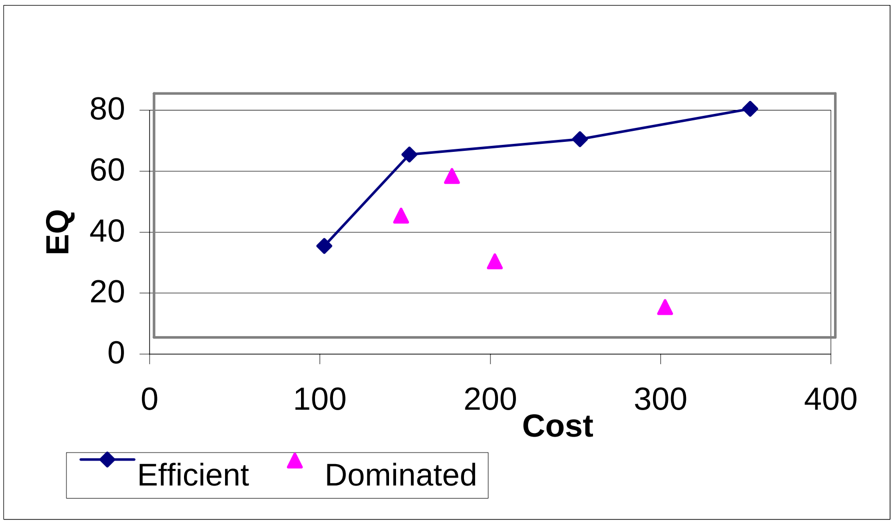



**FIGURE V-2**
**DOMINATED ALTERNATIVES**

IWR-Plan (http://www.pmcl.com/iwrplan/) is software specifically developed by the Institute for Water Resources to provide calculation of the efficient set over a large number of alternatives, as a component of a larger comprehensive methodology of Cost Effectiveness and Incremental Cost Analysis.

## NON-TRADEOFF METHODS OF CHOOSING ALTERNATIVES

Because tradeoff judgments are often difficult and complex, alternative approaches are often carried out at an earlier stage, with easy to use decision-rules that do not require apparent tradeoffs. The following discussion follows Bodily (1985).

Among the non-tradeoff decision rules are:

- Elimination by aspects

  An alternative is selected if it remains after the decision maker has sequentially eliminated alternatives not possessing desired attributes. In many situations people make decisions by through a process of elimination. The decision maker considers one attribute at a time. Alternatives that do not possess a desired aspect are dropped from consideration. Those that possess the aspect are retained. The process continues until all but one alternative remains.

- Lexicographic rules

  Alternatives are rank-ordered according to their scores on a most-important attribute. If alternatives tie on this attribute, they are rank ordered using a second attribute, then a third, and so on, until all ties are broken. This does not fully consider all attributes (there might be very small difference in the first attribute and a large difference in a second attribute for a pair of alternatives, but the small difference will govern, and the rule will never examine the second attribute).

- Conjunctive procedures/satisficing

  Accept an alternative if preset standards or thresholds are met on all attributes. There may be none, one, or more than one satisficing alternative.

- Disjunctive Procedures

  Acceptable an alternative if it scores high enough on at least one attribute.

Lexicographic Rule

“ First, alternatives are rank-ordered according to their scores on a most-important attribute. If alternatives tie on this attribute, they are rank ordered using a second attribute, then a third, and so on , until all ties are broken. ... The lexicographic procedure is easy to use since the decision maker specifies only the order in which the attributes are to be considered. Unfortunately, the rule is often inadequate because it does not fully consider every attribute. Only one attribute is used unless there is a tie.”

Satisficing

“Herbert Simon, Nobel laureate in economics, has suggested that a ‘satisficing’ rule is often used by decision makers. The decision maker searches until finding an alternative that exceeds some aspiration level on each attribute. Like the efficient set, there may be more than one satisficing alternative, but unlike the efficient set, there may be none. Simon asserts that decision makers seldom exhibit optimizing behavior rather than this satisficing behavior. Even though, perhaps, everyone satisfices in routine decisions, it may not be the best approach for important choices, or for those based on formal analysis. The aspiration levels may not be easy to set explicitly. Conceptually, they may not hold up under careful scrutiny because an infinitesimal decline in some attribute level may change an alternative from acceptable to unacceptable.”

Problems with non-tradeoff approaches

Neither the lexicographic nor satisficing approaches allow consideration of the compensating effects of attributes. In other words, a superior performance on one attribute may not compensate for a poor performance on another attribute. The lexicographic rule, satisficing, and a combination of the two avoid tradeoffs among the attributes. Nonetheless, in setting the order of importance of the attributes in the lexicographic rule, or the aspiration levels in the satisficing rule, it is necessary to make very strong preference statements. Because these rules are simple to use, they have their place in practice. However, more robust methods are needed to capture the compensating effects of one attribute for another.”

(Bodily, 1985)

## TRADEOFF METHODS

The drawbacks of the non-tradeoff methods have led to the development of a wide variety of alternative tradeoff-oriented approaches. This is in active area of research and development, with a number of available software tools to assist in exploration of tradeoffs.

## Additive Models

Additive models are among the simplest of the tradeoff methods. The basic approach involves assigning weights to criteria, and development of standard methods of scoring within each criterion. This allows high scores on one criterion to compensate for lower scores on other criteria. For each alternative, a total score is generated based on the criterion weight multiplied by the individual criterion score for that alternative. Under a simple weighting scheme, each individual criterion score must be measured in the same direction (i.e., larger = better). An example additive model problem is provided in Table V-3.

**TABLE V-3**
**Additive Model Example**

| Alternative | Cost Measure | Revised Cost = (300 – Cost) | Environmental Quality Measure | Total Score |
|---|---|---|---|---|
| Criterion Weight | | 0.7 | 0.3 | |
| A | 100 | 200 | 30 | 170 |
| B | 200 | 100 | 25 | 77.5 |
| C | 150 | 150 | 40 | 117 |

Under this particular set of weights and score normalizations, option A is preferred as having the highest total score. Note that this choice is dependent upon both the normalization technique for the cost measure, and the choice of weights. Frequently, each criterion is normalized on a 0 to 100 scale, in an attempt to force all of the preference into the weights.

There are two strong assumptions built into this approach:

1. There is linear value in each criterion - the desirability of an additional unit of any criterion is constant for any level of that criterion;
2. There is no interaction between attributes – they are independent.

A variety of methods have been adopted to deal with the linear value assumption, including the transformation of the scores using the concept of utility functions to translate the score into a utility value, which can vary non-linearly with the score. This is frequently referred to as the Multi-Attribute Utility Theory (MAUT). Other common approaches are the Analytical Hierarchy Process (AHP) or Simple Multiattribute Utility Technique (SMART).

There is no simple method of dealing with the interaction problem. Criteria should be designed to be independent in terms of decision-maker preferences:

Building a set of Criteria

"It is important that the resulting criteria to be used in the decision aid problem have the following basic properties:

- be complete and exhaustive – all important performance attributes deemed relevant to the final solution must be represented by criteria on the list.
- be mutually exclusive : this permits the decision maker to view the criteria as independent entities among which appropriate 'trade-offs' may subsequently be made. This property also helps prevent 'double-counting' through the mutual exclusivity of the criteria.
- be restricted to performance attributes of real importance to the decision problem. This provides a sound starting point for the problem, as the less important/irrelevant/unnecessary criteria can be screened out of the process at the earliest possible stage.

(Rogers, 1999)

The additive method is supported by a number of general software products, including Criterium® DecisionPlus® (www.infoharvest.com) and Expert Choice®, http://www.expertchoice.com/. The approach has also been used as part of the Automated Budget System for the Corps O&M budgeting efforts, in a custom implementation developed by the Institute for Water Resources, and is also a feature of the general purpose IWR-developed MCDM software, available in prototype (2002) through IWR.

## Outranking Methods

Outranking methods or concordance analysis, very popular in Europe where they were originally developed, compare all alternatives pair-wise, and determine those alternatives that are dominant over others. When an alternative A is at least as good as an alternative B for a majority of criteria, and there exists no criterion for which A is substantially less good than B, we can safely say that A outranks B.

One of the attractive features of the outranking methods is the ability to define a preference function for a criterion, allowing a user to express a level at which he/she is indifferent between alternatives on that criterion. For example, for habitat units, it may be desirable to ignore any difference that is less than 1 acre. Thus, two plans that different only by 1 acre in habitat unit would be considered essentially identical. This is a 'fuzzy set' approach to decision-making. A variety of alternative preference functions can be used to express the types of indifference that a decision-maker might have – strict, linear, based on a curve, etc.

The basic thrust of the outranking methods is to compare pairs of alternatives, criterion by criterion, developing a numerical matrix showing the degree to which each alternative is preferred to each other alternative. The numbers in this matrix can then be used to develop an ordering that shows strength of preference between alternatives. The ordering can also display *incomparability* between alternatives, i.e. situations in which it is not unambiguously possible to determine which alternative is to be preferred.

Promethee [**P**reference **R**anking **O**rganization **Meth**od for **E**nrichment **E**valuations]and Electre [from the french Elimination et Choix Traduisant La Realite] are two of the more prominent outranking methodologies, supported by various software techniques, e.g., DecisionLab ( http://www.visualdecision.com), and the previously mentioned IWR-MCDM prototype software.

"[Promethee] is being used more and more frequently, especially for problems of location: hydroelectric stations, stores in a competitive environment, garbage disposal sites,. applications also include financial assessment. The main feature claimed for this method is that it is perfectly intelligible for the decision maker, and the present authors agree that it is indeed one of the most intuitive of multicriterion decision methods." (Pomerol , 2000)

## STRATEGIES FOR DECISION-MAKING UNDER UNCERTAINTY

A fundamental approach to decision-making under uncertainty is to maximize expected value. This, however, amounts to a lexicographic rule, in which the sole criterion is expected value, and has the basic drawback of that rule as noted above. A small difference in expected value between alternatives might be associated with large differences in risk and uncertainty, and thus a risk-averse decision-maker might prefer lower expected value for lower risk.

A risk-averse decision-maker will attempt to minimize the maximum risk (minimize exposure to bad things), and thus choose an alternative that is satisficing on the expected value criterion (i.e., within some acceptable range), but also minimizes the maximum risk. This is an attractive strategy in that it explicitly recognizes the need for examination of uncertainty in the decision-making process, but is relatively simple to display and implement.

## HOW MUCH INFORMATION IS ENOUGH?

In general, it is desirable to have the best estimates of uncertainty in the important decision variables, but this comes at a cost. It may not be possible to reduce the estimates of natural variability but with increased effort, time, and resources (for such things as modeling, data collection, theoretical efforts, and analysis) the knowledge uncertainty can be reduced. There are natural and practical limits to how good an estimate can be obtained for a given variable. In general, efforts at reducing estimates of uncertainty should be directed to those variables that have the greatest impact on the ultimate decision process. If one particular criterion shows high uncertainty, but is not considered especially important in the overall decision process, then the variables that affect the scores on that criterion need not be developed to a high degree of certainty. That is, if uncertainty in a variable will have little impact on the ultimate decision, then the magnitude of that uncertainty is not of primary concern (within limits).

# VI. AN EXAMPLE OF DECISION-MAKING INCORPORATING RISK AND UNCERTAINTY

The general paradigm for the overall process was previously described. In essence, the steps are as follows:

- Use some methodology (such as Monte Carlo simulation) to explore variability in outcomes of alternatives
- Develop a decision matrix of criteria and alternatives
- Reduce the decision matrix by discarding alternatives
- Use a multi-criterion decision method to rank or choose among the alternatives

A hypothetical study example is used to demonstrate the recommended approaches. The example assumes that seven alternatives (A,B,C,D,E,F, and G) are being examined, and each alternative is measured on three criteria:

- Cost
- Benefit
- Environmental Quality (EQ)

In addition, net benefit is derived as Benefit – Cost.

A Monte Carlo simulation tool is assumed to have been developed for the study, and is used to generate, for each of the eight alternatives, 1000 iterations of Cost/Benefit/EQ points. Each such iteration is considered to be a sample from the possible population of outcomes for the alternative. Thus, for alternative A, the first ten iterations are shown in Table VI-1 (units are arbitrary):

**TABLE VI-1**
**ITERATIONS FOR ALTERNATIVE A**

| Iteration # | Cost | Benefit | EQ | Net Benefits |
|---|---|---|---|---|
| 1 | 973.6 | 968.0 | 205.8 | -5.6 |
| 2 | 1050.1 | 1315.5 | 205.2 | 265.3 |
| 3 | 1057.2 | 881.2 | 203.5 | -176.0 |
| 4 | 1110.6 | 961.7 | 191.1 | -148.9 |
| 5 | 987.3 | 1062.7 | 200.8 | 75.3 |
| 6 | 967.8 | 839.3 | 226.9 | -128.5 |
| 7 | 1005.2 | 1050.0 | 184.9 | 44.7 |
| 8 | 950.7 | 1048.3 | 199.7 | 97.5 |
| 9 | 1012.5 | 1098.9 | 186.3 | 86.3 |
| 10 | 995.3 | 1067.0 | 214.2 | 71.6 |

The variability can be displayed in a number of ways, as, for example, a scatter plot of EQ vs. Net Benefits for these 10 points, as shown in Figure VI-1, or a 3-d plot displaying cost, benefit, and environmental quality, as shown in Figure VI-2. These techniques, however, are limited when more criteria or a very large number of iterations are used.

Similar displays could be developed for all of the seven alternatives.

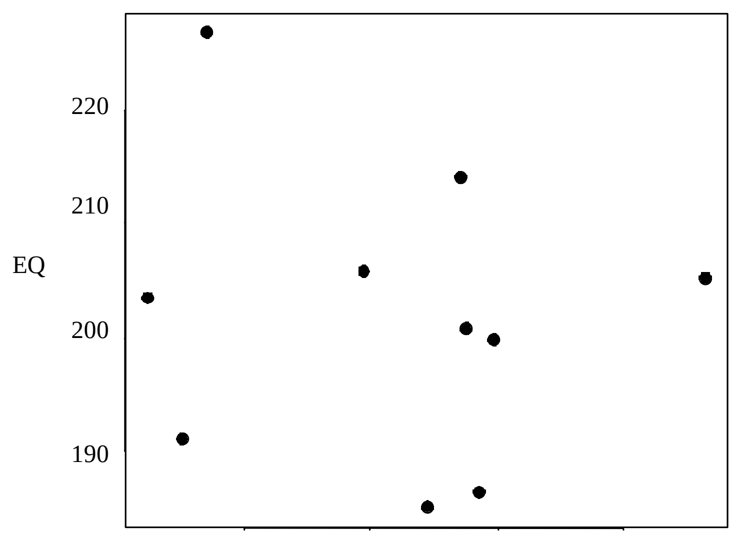



**FIGURE VI-1**
**SCATTER PLOT**

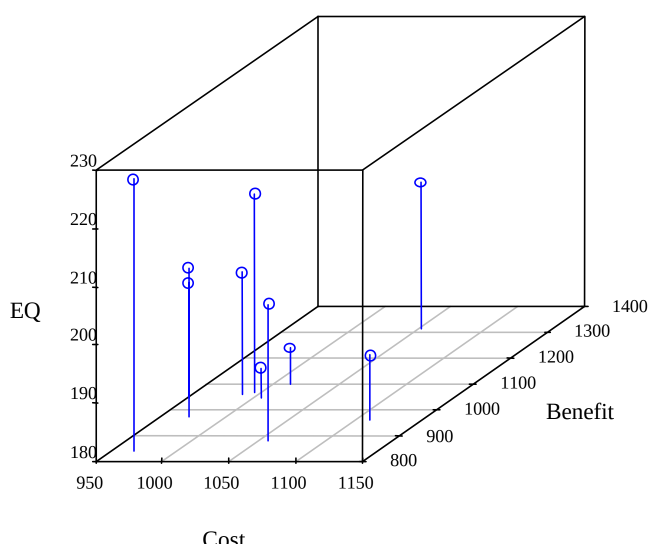



**FIGURE VI-2**
**3D PLOT**

The statistics associated with the Monte Carlo simulation runs for alternatives A through G are shown in table VI-2.

**TABLE VI-2**
**MONTE CARLO SIMULATION STATISTICS A-G**

| Alternative | Mean Cost | Mean Benefit | Mean EQ | Mean Net Benefit |
|---|---|---|---|---|
| A | 1004.0 | 1008.0 | 200.2 | 3.939 |
| B | 880.4 | 815.2 | 180.2 | 14.83 |
| C | 1139.0 | 1195.0 | 189.6 | 55.48 |
| D | 1199.0 | 1234. | 200.8 | 35.65 |
| E | 1100.0 | 1107. | 202.1 | 7.64 |
| F | 999.7 | 1011.0 | 148.1 | 11.17 |
| G | 1151.0 | 1170.0 | 214.5 | 18.91 |

This is the type of data that would typically be presented in a decision-making process, leading to the choice of Alternative C, with the highest mean net benefit. The comparison of EQ score vs. Net Benefits for each of the alternatives is shown in Figure VI-3.

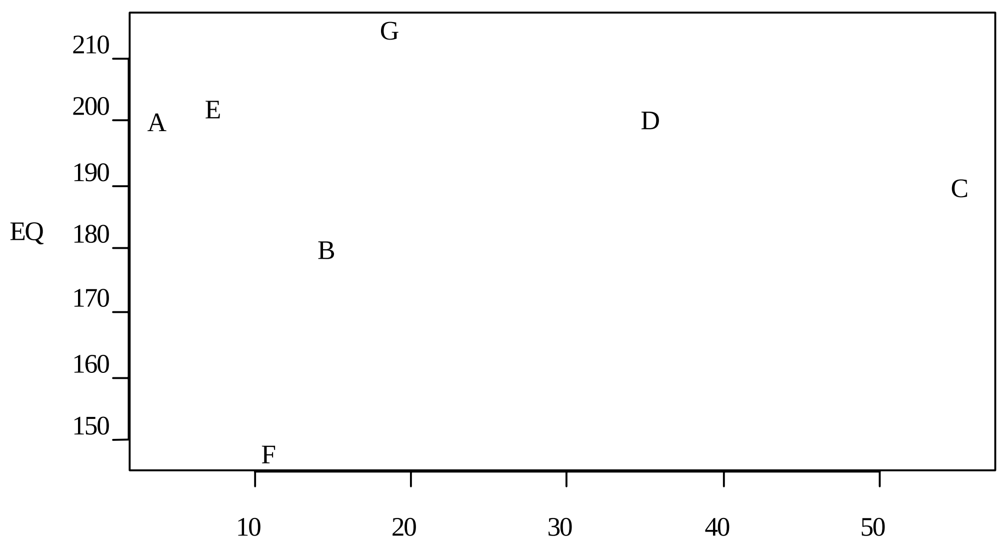



**FIGURE VI-3**
**NET BENEFITS VS. EQ SCORE**

Under the concept of efficiency, alternative G gives the highest possible EQ score, and alternative C gives the highest net benefits. Examining only the criteria of Net Benefit and EQ score, Alternatives A, E, B, and F are considered to be inefficient, in that it is possible to choose an alternative with higher net benefits and better EQ score for each of these alternatives. This does not, however, take into account the variability associated with each of these alternatives, only the mean value.

More information is revealed, however, by examining the distributions associated with each of the alternatives. The 1000 iterations for each alternative, when analyzed statistically, give the following additional summary data, shown in Table VI-3, on which to base decisions.

**TABLE VI-3**
**ADDITIONAL SUMMARY DATA**

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| **Cost** | | | | | | | |
| **Min** | 860.3 | 296.3 | 733.8 | 938.9 | 496.2 | 861.0 | 1075 |
| **1st Qu** | 970.5 | 699.0 | 1041.0 | 1143.0 | 967.1 | 967.7 | 1130 |
| **Median** | 1003.0 | 799.0 | 1141.0 | 1200.0 | 1102.0 | 1001.0 | 1150 |
| **Mean** | 1004.0 | 800.4 | 1139.0 | 1199.0 | 1100.0 | 999.7 | 1151 |
| **3rd Qu** | 1037.0 | 899.1 | 1236.0 | 1251.0 | 1236.0 | 1032.0 | 1170 |
| **Max** | 1188.0 | 1310.0 | 1599.0 | 1431.0 | 1735.0 | 1188.0 | 1261 |
| **Benefit** | | | | | | | |
| **Min** | 711.6 | 740.1 | 267.7 | 946.8 | 376.7 | 935.4 | 1002 |
| **1st Qu** | 939.6 | 798.7 | 1042.0 | 1178.0 | 973.8 | 994.4 | 1135 |
| **Median** | 1010.0 | 816.0 | 1194.0 | 1237.0 | 1112.0 | 1011.0 | 1170 |
| **Mean** | 1008.0 | 815.2 | 1195.0 | 1234.0 | 1107.0 | 1011.0 | 1170 |
| **3rd Qu** | 1073.0 | 831.7 | 1345.0 | 1290.0 | 1239.0 | 1027.0 | 1204 |
| **Max** | 1346.0 | 908.3 | 1909.0 | 1464.0 | 1694.0 | 1096.0 | 1320 |
| **Net Benefit** | | | | | | | |
| **Min** | -375.200 | -480.20 | -806.00 | -374.30 | -818.7000 | -161.10 | -168.70 |
| **1st Qu** | -69.170 | -86.44 | -129.90 | -42.96 | -182.4000 | -25.06 | -19.97 |
| **Median** | 7.068 | 13.29 | 56.31 | 37.06 | 0.7986 | 11.62 | 19.81 |
| **Mean** | 3.939 | 14.83 | 55.48 | 35.65 | 7.6410 | 11.17 | 18.91 |
| **3rd Qu** | 79.270 | 117.10 | 234.50 | 111.80 | 214.0000 | 47.69 | 60.33 |
| **Max** | 398.900 | 468.50 | 1061.00 | 350.20 | 857.3000 | 196.90 | 211.90 |
| **EQ** | | | | | | | |
| **Min** | 145.4 | 92.3 | 46.64 | 64.65 | -41.89 | -75.28 | -75.77 |
| **1st Qu** | 187.1 | 159.1 | 160.60 | 175.50 | 148.40 | 107.50 | 155.60 |
| **Median** | 200.0 | 180.0 | 189.70 | 199.60 | 201.00 | 148.30 | 218.60 |
| **Mean** | 200.2 | 180.2 | 189.60 | 200.80 | 202.10 | 148.10 | 214.50 |
| **3rd Qu** | 213.7 | 199.8 | 218.50 | 227.40 | 252.60 | 186.00 | 272.50 |
| **Max** | 264.1 | 271.1 | 305.50 | 338.10 | 461.40 | 371.30 | 480.60 |

The standard deviation is a common measure of variability. The standard deviation can be added to the display of the mean values associated with each alternative, shown in Table VI-4.

**TABLE VI-4**
**MEAN VALUES ASSOCIATED WITH EACH ALTERNATIVE**

| Alternative | Mean Cost | SD Cost | Mean Benefit | SD Benefit | Mean EQ | SD EQ | Mean Net Benefit | SD Net Benefit |
|---|---|---|---|---|---|---|---|---|
| **A** | 1004.0 | 49.05 | 1008.0 | 97.83 | 200.2 | 19.62 | 3.939 | 110.94 |
| **B** | 880.4 | 145.76 | 815.2 | 24.61 | 180.2 | 29.26 | 14.83 | 148.05 |
| **C** | 1139.0 | 145.75 | 1195.0 | 233.92 | 189.6 | 40.23 | 55.48 | 278.26 |
| **D** | 1199.0 | 81.88 | 1234. | 83.29 | 200.8 | 39.86 | 35.65 | 110.66 |
| **E** | 1100.0 | 202.63 | 1107. | 200.99 | 202.1 | 77.42 | 7.64 | 282.30 |
| **F** | 999.7 | 49.54 | 1011.0 | 25.22 | 148.1 | 60.24 | 11.17 | 55.23 |
| **G** | 1151.0 | 29.76 | 1170.0 | 51.75 | 214.5 | 81.95 | 18.91 | 60.29 |

Examination of this information shows that Alternative C, with the highest mean net benefit, also has high variability in benefits and thus in net benefits. The minimum net benefit of the 1000 iterations was –806, and the 1st quartile value shows that 25 percent of the iterations for Alternative C have net benefits less than 129.9

There are a variety of graphical ways of displaying this information. The box plot in Figure VI-4 shows the ranges associated with each of the alternatives, in this case demonstrating that there is a much wider range of net benefits for alternatives C and E.

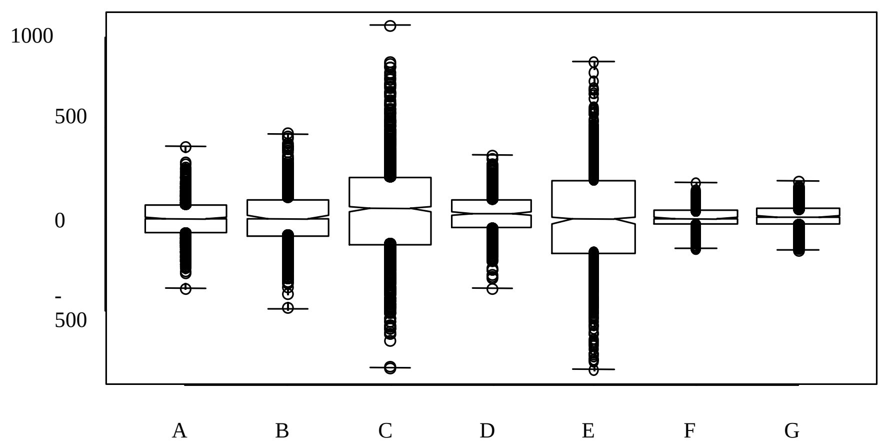



**FIGURE VI-4**
**NET BENEFIT BOX PLOT**

The following box plot in Figure VI-5 shows that alternatives A, B and F are significantly less costly than the other alternatives.

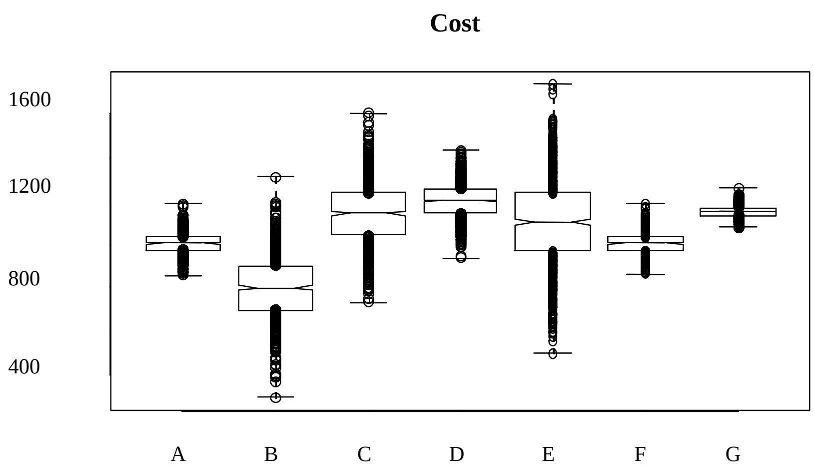



**FIGURE VI-5**
**COST BOX PLOT**

The probability density function displays the distribution completely, as shown in Figure VI-6 below, in which the wide range of net benefits associated with alternative C is clearly seen.

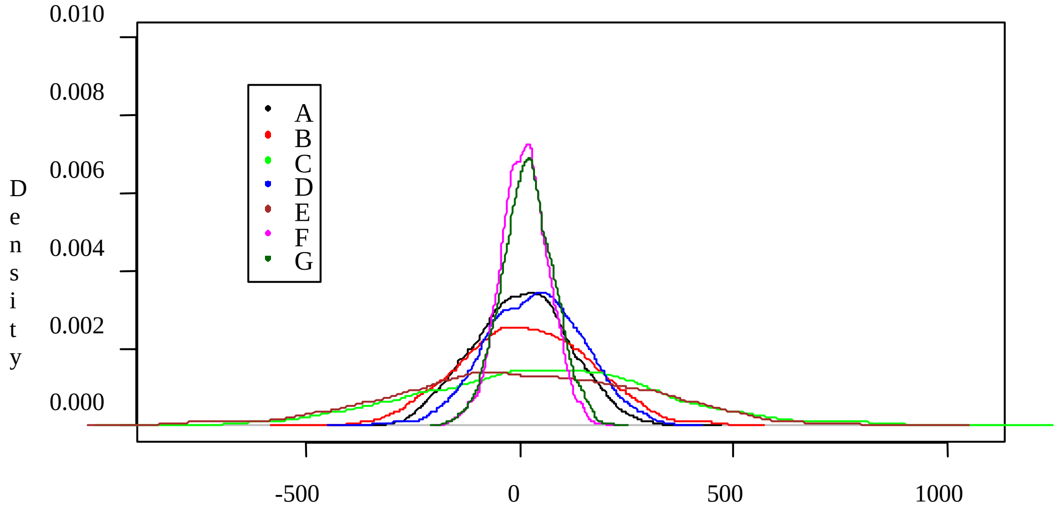



**FIGURE VI-6**
**PROBABILITY DENSITY FUNCTION**

The cumulative density function (CDF) can be used to show stochastic dominance, that is, which alternatives are better over the complete range of probabilities. When the CDF curves do not cross each other, one alternative is said to dominate the other. Here, Alternative B is clearly the lowest cost over almost the complete range of probabilities.

The variability can be displayed in a scatter plot Figure VI-7 of Net Benefits vs. EQ Score for each alternative:

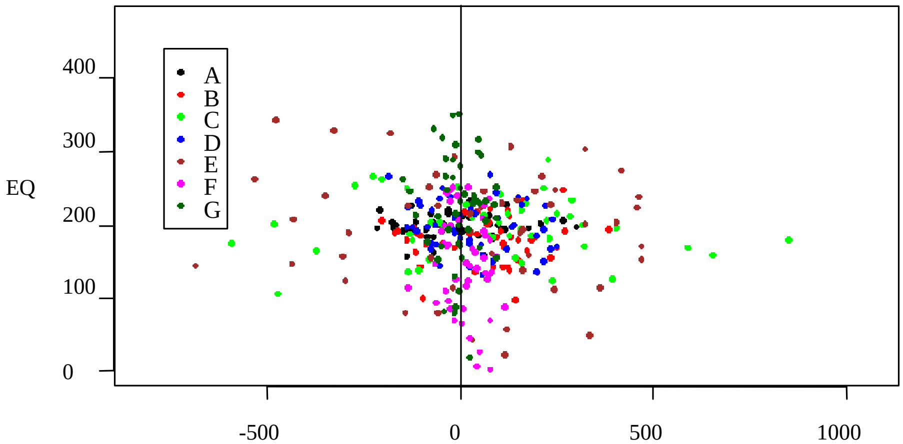



**FIGURE VI-7**
**EQ vs. NET BENEFIT**

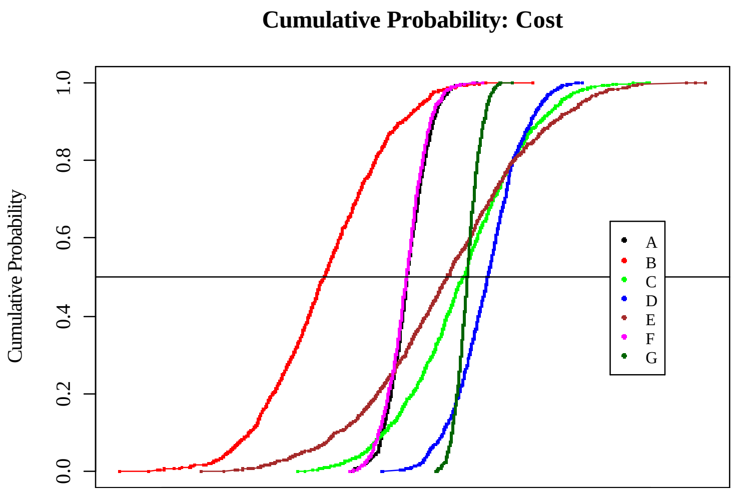



**Figure VI-8**
**CUMULATIVE PROBABILITY: COST**

The previous plot, in Figure VI-7, showing the mean of EQ vs. Net Benefit can be extended to show the 50 percent probability variation around each mean, as shown in Figure VI-8, based on the quartile information presented earlier. In Figure VI-9 the range of Net Benefits and EQ is plotted as a cross-hair of variable length centered at the mean:

The foregoing examination suggests that, while Alternative C has the highest net benefits, it is also relatively risky. Thus, a decision-maker who is risk-averse might wish to consider other possibilities.

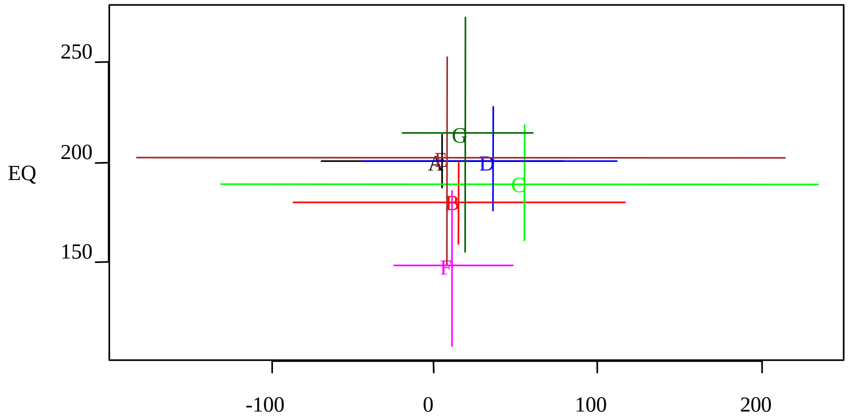



**FIGURE VI-9**
**NET BENEFITS vs. EQ SCORE WITH 50% RANGES**

The next step in evaluation is to limit the range of alternatives. As noted previously, this can be done by discarding 'inefficient' alternatives, in this case alternatives A, B, E, and F. If cost is considered to be a separate criterion, then retention of alternative B might be worthwhile, as it is the least costly. Other methods of reducing the decision matrix also exist, such as setting a minimum level of EQ score of 190, which would eliminate alternatives B, C, and F. For the example, assume that the inefficient alternatives have been eliminated (recalling that this is based exclusively on the mean value, and this can frequently rule out alternatives that are in effect

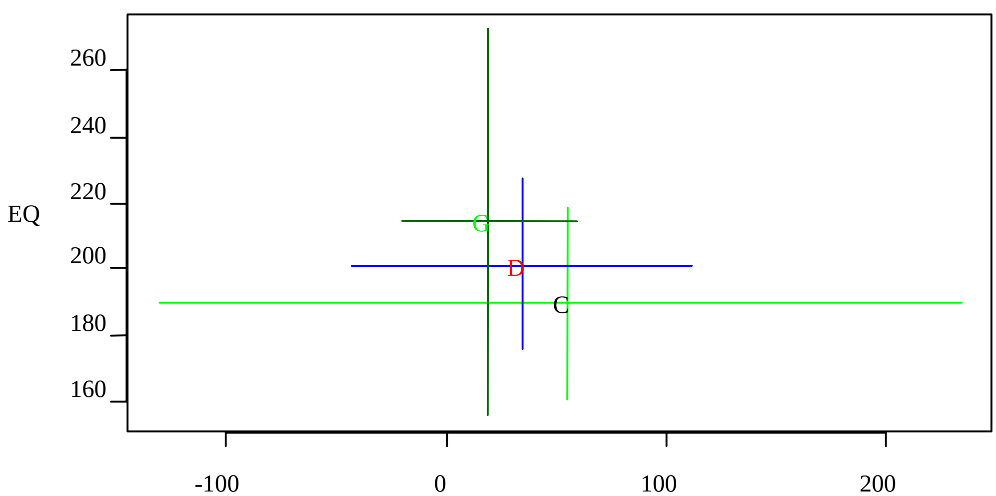



**FIGURE VI-10**
**NET BENEFITS vs. EQ SCORE WITH 50% RANGES**

worthy of further examination). This leads to the FIGURE VI-10 net benefits vs. EQ score plot with range, showing only the remaining alternatives G, D and C.

At this point, the basic decision matrix as shown in Table VI-5 has been reduced to:

**TABLE VI-5**
**BASIC DECISION MATRIX**

| Alternative | Mean Cost | SD Cost | Mean Benefit | SD Benefit | Mean EQ | SD EQ | Mean Net Benefit | SD Net Benefit |
|---|---|---|---|---|---|---|---|---|
| **C** | 1139.0 | 145.75 | 1195.0 | 233.92 | 189.6 | 40.23 | 55.48 | 278.26 |
| **D** | 1199.0 | 81.88 | 1234. | 83.29 | 200.8 | 39.86 | 35.65 | 110.66 |
| **G** | 1151.0 | 29.76 | 1170.0 | 51.75 | 214.5 | 81.95 | 18.91 | 60.29 |

Here, each criterion is measured by the mean and standard deviation, with an assumed normal distribution. To simplify even further, the decision-maker can focus exclusively on EQ and Net Benefit, shown in Table VI-6.

**TABLE VI-6**
**EQ AND NET BENEFITS**

| Alternative | Mean EQ | SD EQ | Mean Net Benefit | SD Net Benefit |
|---|---|---|---|---|
| **C** | 189.6 | 40.23 | 55.48 | 278.26 |
| **D** | 200.8 | 39.86 | 35.65 | 110.66 |
| **G** | 214.5 | 81.95 | 18.91 | 60.29 |

Absent the requirement to examine only the NED plan, there is no clear choice between these three alternatives on the EQ and NED criteria. Each alternative gives a different mix, and tradeoffs are necessary in order to choose a final plan.

In the above decision matrix there are two criteria, (EQ and Net Benefit), with the distribution of each criterion specified (in this case, by the mean and standard deviation). Two approaches are possible, depending upon the tools available: if the decision tool can support explicit definition of distributions, then this information can be carried forward directly; alternatively, risk measures can be considered as separate ‘pseudo-criteria’, that is, the decision maker considers the expected value and risk measure separately, and expresses preferences for each.

A number of methods are available to evaluate the respective plans using tradeoff analysis. As noted previously, most of these methods require that a preference between criteria be stated, usually in the form of weights associated with each criterion, to express the level of importance of each criterion in the decision process. At that point, an additive or outranking approach can be used. The additive approach is used to normalize the measures associated with each criterion, and then, in essence, develop a composite score for each alternative based on the decision matrix and the weights.

Criterium Decision Plus 3.0 software from InfoHarvest is one of the software tools that can be used for additive analysis and is used in the example example. The first step in using this, and related tools, is to develop an appropriate hierarchy, as displayed in Figure VI-11:

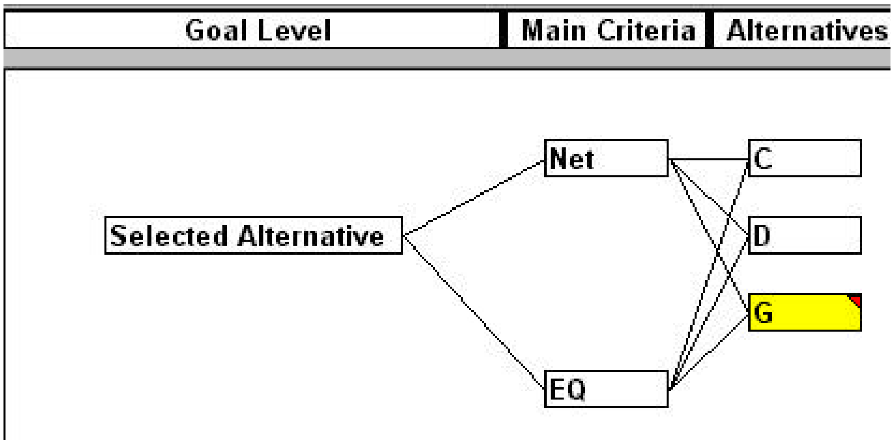



**FIGURE VI-11**
**CRITERIUM DECISION PLUS HIERARCHY DEVELOPMENT SCREEN**

The distribution associated with each criterion and alternative can be entered as seen in the example in Figure VI-12:

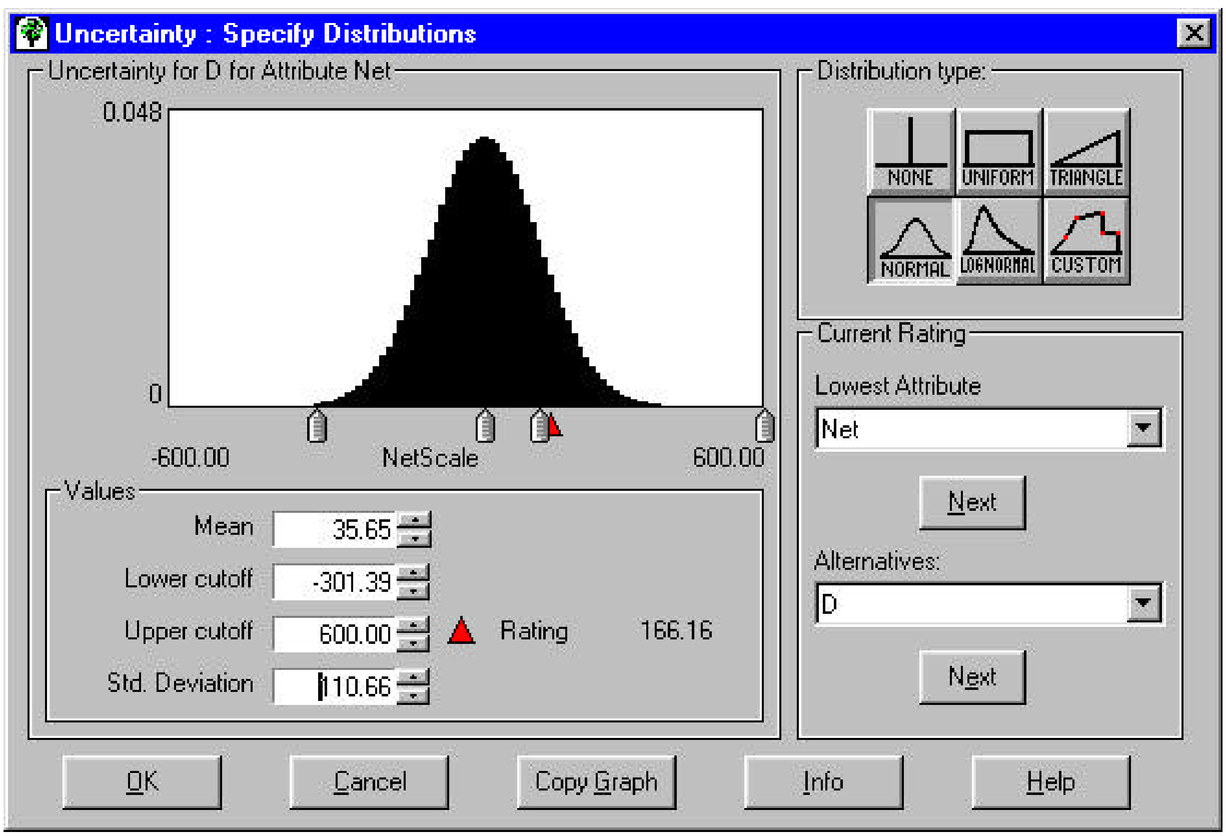



**FIGURE VI-12**
**SPECIFYING DISTRIBUTIONS**

When such information is entered for each alternative and criterion, the distribution of decision scores is calculated. A visualization of this is provided as Figure VI-13.

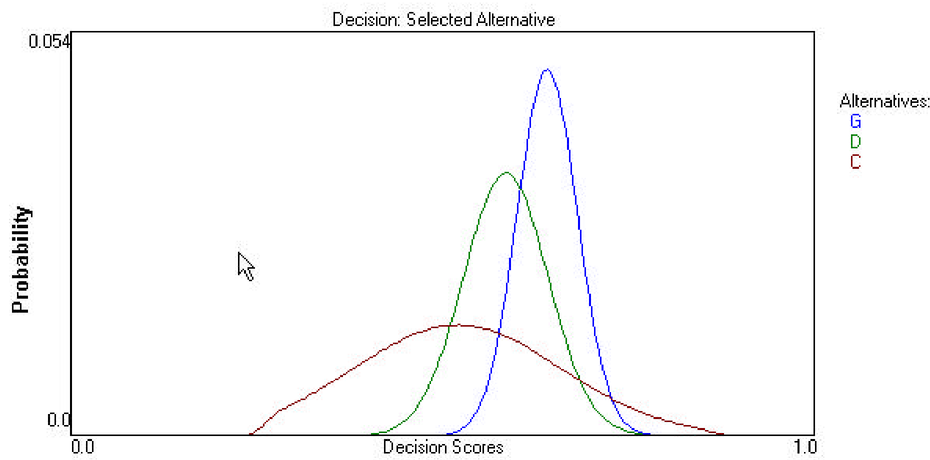



**FIGURE VI-13**
**DECISION SCORE DISTRIBUTION**

This in turn provides a display of the percent of time that each alternative is better than all of the others as shown in Figure VI-14.

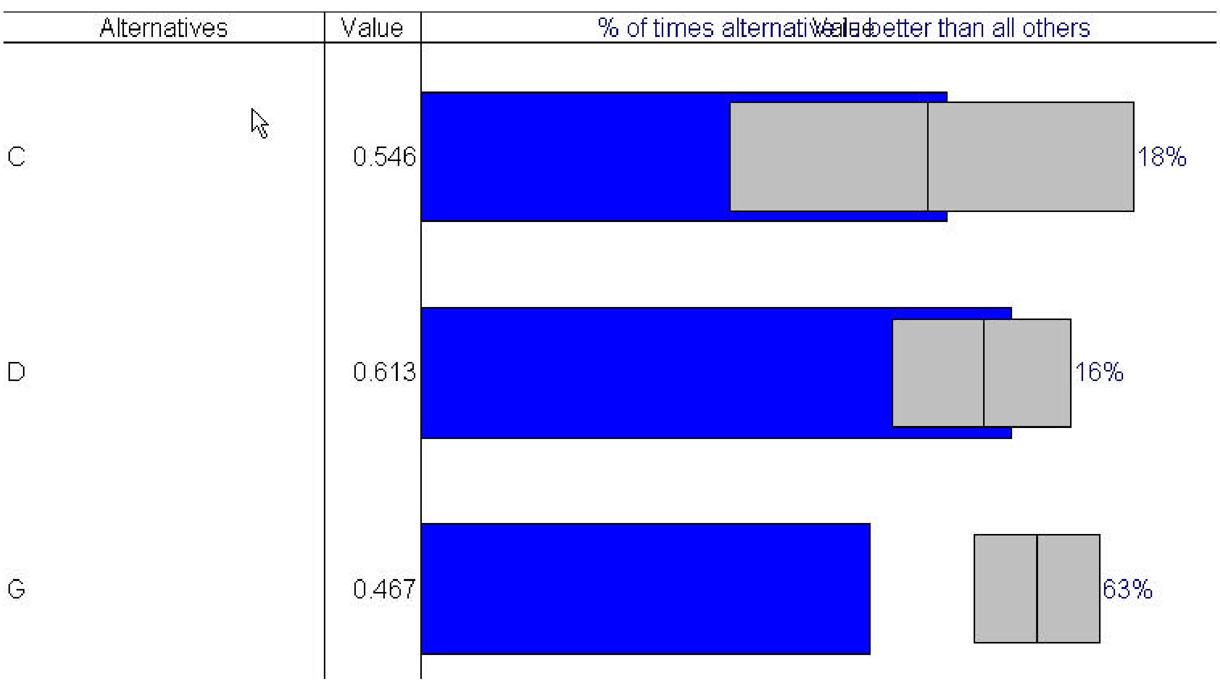



**FIGURE VI-14**
**PERCENT OF TIME ALTERNATIVE IS BETTER THAN ALL OTHERS**

This method makes direct use of the distributions of the criteria. It does not, however, allow the decision maker to express an explicit preference relating to the risk levels associated with each alternative and criterion. By expressing the risk measure as a separate pseudo-criterion, this can be done.

As an example of such an approach, the decision matrix above can be normalized to a 'desirability score' on a scale of 0 to 100, by setting the lowest value in each column to 0, and the highest to 100, and interpolating in between. Note that this linear interpolation may not truly express the desirability preference of a decision-maker, which may be non-linear. The scoring can use any function to translate the actual measure of each alternative and criterion to a score—a linear function is used here for simplicity. This results in a scoring matrix as shown in Table VI-7.

**TABLE VI-7**
**SCORING MATRIX**

| | Normalized EQ Score | Normalized SD EQ Score | Normalized Mean Net Benefits Score | Normalized SD Net Benefits Score |
|---|---|---|---|---|
| C | 0 | 0.879069 | 100 | 100 |
| D | 44.97991968 | 0 | 45.77523 | 23.10868 |
| G | 100 | 100 | 0 | 0 |

It is reasonable to assume the Corps and local decision-makers are generally risk-averse. The desire to avoid large damages and potential loss of life due to flooding is an example. A risk-averse decision-maker will prefer lower probability of undesirable outcomes. Here, we are using standard deviation as a surrogate measure for risk (risk should more properly be measured in terms of probabilities of a particular alternative and criterion being above or below a designated threshold value but for purposes of the example the surrogate measure is used), and a risk-averse decision-maker would likely prefer more certainty in estimates of outcomes. Thus, a risk-averse decision-maker would favor lower standard deviations for each criterion. In the example above, the risk-averse decision-maker would like to minimize the SD EQ and SD Net Benefits, and maximize the EQ and Net Benefits. As noted previously, the normal Corps approach might dictate selection of alternative C, which has the highest net benefit score. This alternative also has the highest risk associated with net benefits, and the lowest EQ score.

The fundamental tradeoff questions are then:

- how important is minimizing the risk associated with net benefits compared to achieving higher net benefits?
- what tradeoffs, if any, should be considered between EQ and Net Benefits?

The preferences placed on these values by decision-makers can be reflected in weight assignments to criteria.

The hierarchy now looks as follows in Figure VI-15:


**FIGURE VI-15**
**CDP HIERARCHY OF EXAMPLE PROBLEM**

Here, the ultimate goal of selecting a plan alternative is based on the four criteria defined. The three plan alternatives are scored as above against each of the criteria, and weights are assigned to the criteria themselves. The software then develops a ranking and score of each alternative, and has capability to display the sensitivity of the ranking to the weights. In tradeoff analysis, weights are typically assigned on some range, such as 0 to 1, 0 to 100, etc. The software will typically normalize all the weights so that the sum of all weights is equal to 1.0.

Depending upon the weights assigned to the criteria, different outcomes result. In the example below, weights are assigned in a range of 0 to 100 for each criterion, under three different scenarios. The resulting scores are those calculated by CDP 3.0, with the highest score being the most preferred alternative.

Table VI-8 provides an example of weights and their effect on alternative scores.

| TABLE VI-8 WEIGHTS AND SCORES | | | |
|---|---|---|---|
| **Criterion Weight** | **1** | **2** | **3** |
| **Net** | 75 | 75 | 60 |
| **SD Net** | 25 | 50 | 40 |
| **EQ** | 25 | 25 | 40 |
| **SD EQ** | 10 | 10 | 10 |
| **Alternative Score** | | | |
| **C** | 0.629 | 0.531 | 0.466 |
| **D** | 0.554 | 0.588 | 0.575 |
| **G** | 0.37 | 0.469 | 0.533 |

Under scenario 1, net benefits are weighted highly (75), with less importance given to the standard deviation of net benefits and to EQ. Alternative C is preferred. If the weight associated with standard deviation of net benefits is increased (greater degree of preference for risk aversion), as in scenario 2, alternative D becomes slightly preferable. As well, if the import associated with EQ is increased and Net Benefits decreased (scenario 3), alternative D is ranked 1st, with alternative G, last in all the other scenarios, now ranked 2nd. A visualization of the sensitivity of alternatives to weight assignment is shown in Figure VI-16.

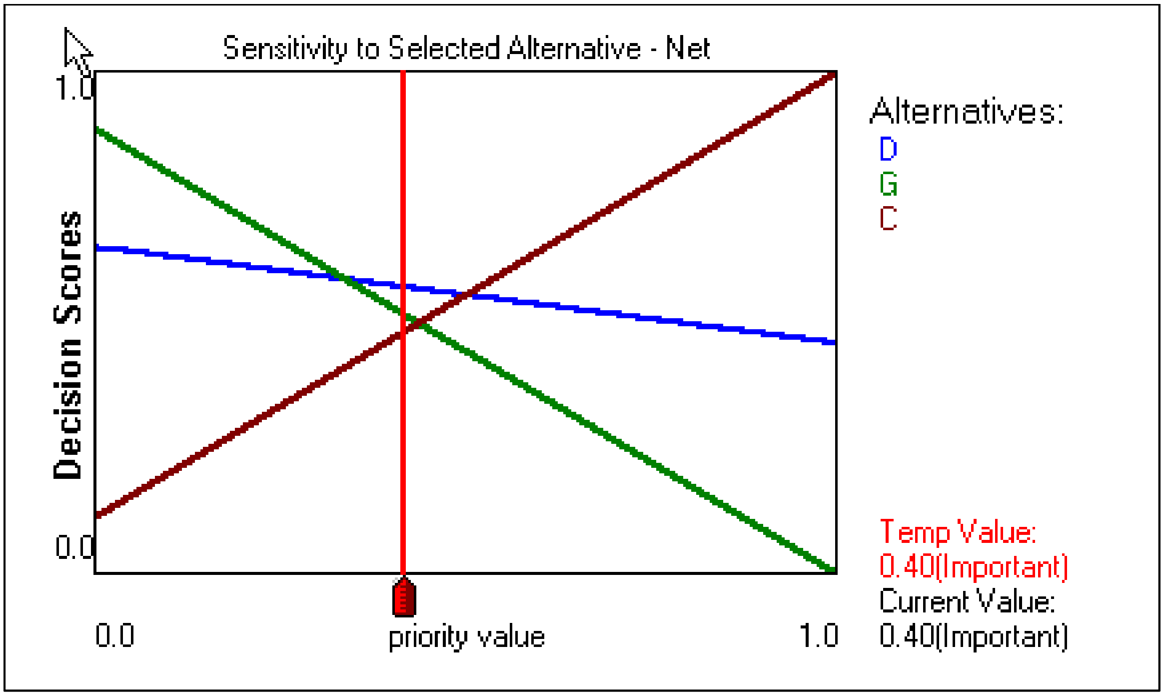



**FIGURE VI-16**
**SENSITIVITY TO SELECTED ALTERNATIVE-NET**

Tools such as CDP allow for a variety of types of examination of sensitivity to weights and ratings. The graphic shows how the selection of a given alternative depends upon the priority assigned to Net Benefits (all other criterion weights being held constant), for the weights assigned in Scenario 3 above. Moving the weight assigned to Net Benefits in a narrow range can result in the preference for any of the three alternatives. Tools such as this can be used to explore the impact of different preference structures, as expressed in the weights. Because such tradeoff analyses can be highly sensitive to the criterion weights, it is essential to examine this issue as part of the ultimate decision process. Many of the software tools that support weighted analysis have capabilities of displaying sensitivity of outcomes to selection of criterion weights.

Other decision aid tools are available, providing different capabilities. The outranking methods of Electre and Promethee allow the user to specify a level of indifference or preference between certain values of criteria, as well as weighting the criteria. For example, a decision-maker may consider that a difference in EQ measures of 20 is insignificant, but a difference of 50 is important. According to traditional Corps planning approaches, an alternative with net benefits (NED) of $1,000,000 is strictly preferred to one whose net benefits are $999,000 (assuming equal costs) but in fact most decision-makers would consider this difference to be insignificant. The outranking methods allow this kind of preference structure within a criterion to be taken into account in determining preference between alternatives.

Promethee uses the concept of *flows* to determine rankings between alternatives. Three kinds of multicriteria preference flows (positive, negative, and net) are used. They provide three ways to rate alternatives, and are the basis of the developed rankings. Positive and negative flows are scored from 0 to 1, while the net flow is scored from –1 to 1.

The positive flow of an alternative measures the extent by which that alternative is preferred to the other alternatives in the decision matrix. A value of 0 indicates that the alternative is not preferred (based on the criteria) to any other one, while a value of 1 indicates that it is preferred to all of the others. The larger the value, the more preferred is the alternative.

The negative flow is the converse of this, and measures the degree to which other alternatives are preferred to a given alternative – a value of 0 indicates that no alternative is preferred, while a value of 1 indicates that all other alternatives are preferred. The smaller the value of negative flow, the better the alternative. When the two sets of flows are used together, a preference display of all alternatives can be developed. This is not necessarily a strict ranking of all alternatives – some alternatives may be considered as equivalent, and others as not properly compared under the stated preference structures.

The net flow combines the positive and negative flows a single rating, defined as the difference between positive and negative flow for the alternative. The best alternatives have positive values of net flow while the worst ones have negative values. The larger the net flow, the better the alternative. The net flow is used to enforce a strict ranking of alternatives when that is desired, but tends to obscure some of the complexities of the problem.

Outranking methods take into account the fact that preference is not strict, but fuzzy.

Proponents of outranking methods point to important features of the approach:

- the ability to model more complex preference and indifference structures easily
- outranking avoids oversimplifications and mathematical anomalies that can be associated with the additive weighting approach
- the fact that the procedures do not force a preference where none exists, and reveal situations where alternatives are not comparable based on the stated criteria and preferences

Decision Lab software (Visual Decisions, Inc.) was used to explore an outranking approach for the same decision matrix used in Table VI-8, in Table VI-9:

Here, for the Mean EQ and SD Net criteria, a preference function that is structured as follows is used. If the two alternatives differ on the criterion by the amount of the preference threshold, then the one is strictly preferred to the other (preference value of 1). If they differ only by the indifference threshold, then there is no preference between them (preference value of 0), and the preference is intermediate (between values of 0 and 1). The preference function used in this example is shown in Figure VI-17.

**TABLE VI-9**
**DECISION LAB SOFTWARE DECISION MATRIX**

| | Mean EQ | SD EQ | Mean Net | SD Net |
|---|---|---|---|---|
| **Min/Max** | **Maximize** | **Minimize** | **Maximize** | **Minimize** |
| **Weight** | 40.0000 | 10.0000 | 60.0000 | 40.0000 |
| **Preference Function** | Level | Strict | Strict | Level |
| **Indifference Threshold** | 20.0000 | - | - | 10.0000 |
| **Preference Threshold** | 50.0000 | - | - | 100.0000 |
| **C** | 200.8000 | 39.8600 | 35.6500 | 110.6600 |
| **D** | 189.6000 | 40.2300 | 55.4800 | 278.2600 |
| **G** | 214.5000 | 81.9500 | 18.9100 | 60.2900 |

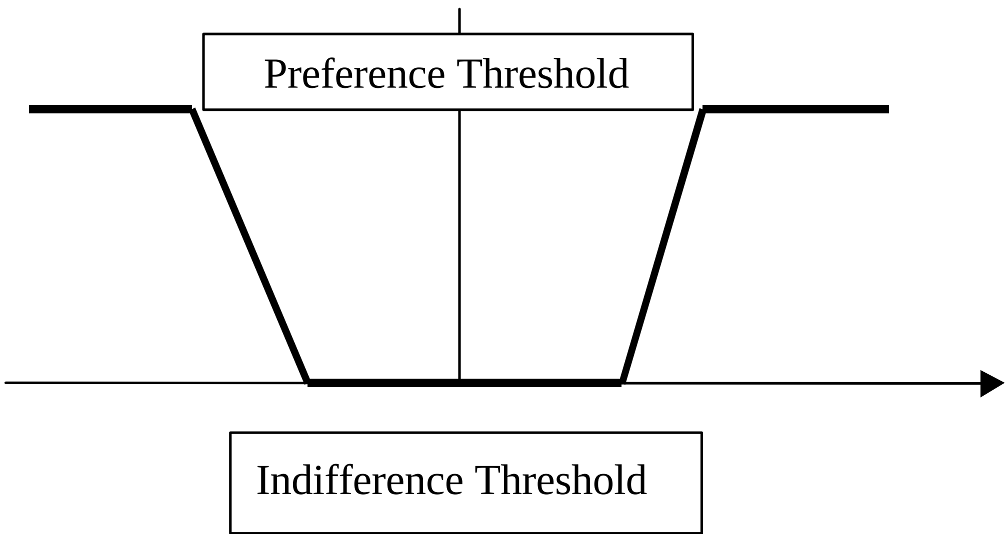



**FIGURE VI-17**
**PREFERENCE FUNCTION**

The flows are calculated as shown in Table VI-10.

**TABLE VI-10**
**CALCULATED FLOWS**

| | Positive Flow | Negative Flow | Net Flow |
|---|---|---|---|
| **C** | 0.4333 | 0.3667 | 0.0667 |
| **D** | 0.4000 | 0.2667 | 0.1333 |
| **G** | 0.2667 | 0.4667 | -0.2000 |

Ranking based on net flows gives D as the preferred alternative, but there is conflicting information when examining the positive and negative flows. The positive flow shows that C and D are both preferable to G, and the negative flows show that C and D are also preferable to G (smaller is better for negative flows). However, C is slightly better on positive flows than D, and D is somewhat better than C on negative flow. Thus, neither C nor D dominate, although it is clear that both C and D are preferred to G (on both the positive and negative flows). The net flow can be used to force tie-breaking, giving a final ranking, but this hides the fact that C and D are reasonably close.

With any of the tradeoff methods, there is clearly an opportunity to explicitly set a risk preference and incorporate that risk preference in the decision process. In the example, the less risky alternative D is compared to the higher yielding but more risky alternative C by explicitly adding the standard deviation of net benefits as a decision criterion, and attempting to minimize it. Under many preference scenarios, C becomes a good choice as compared to D.

# VII. SUMMARY

A basic framework for making decisions under risk and uncertainty has been proposed. The overall goal is to demonstrate that it is both possible and necessary to go beyond simply displaying the expected value associated with alternatives. Rather, explicit quantification and discussion of risk is important and appropriate.

A Monte Carlo simulation or other analytical tool can be used to generate information about the statistical distribution of costs, benefits, and outcomes for a variety of plans, developing information that is more comprehensive than averages or expected values. This information can then be carried forward in the decision process, summarized mathematically and visualized in a number of ways to explore the degree of risk associated with each alternative. The concepts of efficiency and dominance are used to limit the number of alternatives where possible. The problem is then framed as a multi-criterion decision problem, with risk explicitly incorporated. A variety of decision aids are available to assist in determining preferences between the remaining alternatives. It is important to note that framing the problem as a multi-criterion decision problem does not dictate a single solution – rather, it defines an orderly process to reveal the advantages of different alternatives, and allows for the explicit incorporation of risk as one of the decision factors.

Within the Corps there is ample experience, knowledge, skill, and access to tools and techniques to perform the needed analyses to carry forward estimates of risk and uncertainty into the decision-making process. At times, as in the evaluation of major rehabilitation for Hoover Dike by the Jacksonville District, the explicit examination of risk can be an asset to project justification. The basic analytical approaches are already being carried out at the plan evaluation stage, and all that is needed is to carry the findings forward through comparison and plan selection. Simple measures, and simple techniques, can be used to display the risk-associated impacts of alternative plans, allowing an explicit choice to be made that incorporates these impacts. While explicit Corps guidance does not demand that this approach be taken, the general philosophy is certainly that of encouraging greater use of risk-based approaches.

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# *APPENDIX A*

# *BASIC RISK CONCEPTS*

# APPENDIX A: BASIC RISK CONCEPTS

The language, concepts and techniques of risk analysis are frequently unfamiliar to analysts and decision-makers. The terminology is used in many different ways. The language of quantitative risk analysis is that of probability and statistics, unfamiliar to many. Techniques of risk analysis are highly dependent upon computer-based approaches. Adding risk issues to the analysis process significantly expands the arenas of examination. There are few general solutions to “risk problems”—rather, each class of problem (e.g., hydropower, deep-draft navigation, etc.) has its own set of uncertain parameters and interactions, which must be identified and quantified on a case-by-case basis.

On the positive side, risk analyses provide new perspectives on problems, and add significantly to the insights about the true nature of problems and possible solutions. Qualitative risk analysis does not necessarily require detailed mathematical- or data-intensive studies, and can provide good insights for screening alternatives and making routine decisions. Risk analysis is generally seen as a more accurate presentation of decision issues than simpler deterministic approaches can provide. There are many well-accepted techniques and tools for performance of risk assessment, in a wide variety of arenas. The basic principles are relatively straightforward, and are not overly complex. Decision-makers and analysts who familiarize themselves with these concepts will be better able to assess and carry out risk-based studies, and make informed decisions when risks are properly presented.

## WHAT IS RISK?

There are various definitions of risk. It is a subject of much discussion, and there is no general agreement. A workable, usable definition of risk is that risk is a characteristic of a situation, action, or event in which:

- A number of outcomes are possible
- The particular one that will occur is uncertain
- At least one of the possibilities is undesirable

More simply, risk is the chance of something bad happening. Uncertainty is a characteristic of a situation in which:

- A number of possibilities exist
- We do not know which of them has occurred or will occur

Uncertainty exists because of:

- Natural Variability: Nature is random (at our level of view).
- Knowledge Gaps: Lack of knowledge, time or resources. Our knowledge, models, analysis techniques and data are not perfect. Our estimates of parameters for models are not exact.

## RISK ANALYSIS FRAMEWORK

Risk analysis is a tool to aid decision-making. The basic framework for considering risk analysis in the business programs of the Civil Works Program consists of three parts—risk assessment, risk management and risk communication, all of which are seen as overlapping tasks:

### Assessment

- What can go wrong?
- How can it happen?
- How likely is it to happen?
- What are the consequences?

### Management

- What questions should the risk assessment answer?
- What can be done to reduce the impact of the risk described?
- What can be done to reduce the likelihood of the risk described?
- What are the trade-offs of the available options?
- What is the best way to address the described risk?

### Communication

- With whom do you communicate?
- What do people know? How do they know it? What do they want to know?
- How do you get both the information you need and the information others have?
- How do you convey the information you want to communicate?
- When do you communicate?

## GENERAL RISK ASSESSMENT METHODOLOGY

Risk assessment is often carried out in a "systems framework," making use of mathematical models to describe the decision problem. The steps are generally as follows:

- Define the System
  - = Boundaries
  - = Variables to be considered
  - = Relationships between variables
  - = Output measures of interest

- Choose variables that are treated as uncertain
- Characterize uncertainty in each such variable
- Determine the consequences of the combined uncertainty in the variables on the uncertainty in the output measures
- Display the results

Tools such as *decision trees*, *influence diagrams*, and *Monte Carlo simulation* assist in the definition and description of the system, and in the calculation of the results. Generally, computer techniques are necessary to handle the large number of possibilities that result from the combinations of the uncertain variables.

## THE DESCRIPTION OF RISK

There are a number of ways of talking about risk. Each method adds something to the characterization. Linguistic descriptions use words rather than numbers. A risk can be compared against other risks with which we have a more intuitive familiarity. Numerical measures of probability and statistics are apparently precise means of characterizing risk. Shorthand measures can be used (mean, standard deviation), or one can develop a more complete description of the probability distribution.

### Linguistic

> "All too often, we choose words to express the results of risk assessment that have negative connotations. The word risk itself connotes a feeling of danger, insecurity, and precariousness"
>
> (Cothern and Marcus, 1987)

- risky
- dangerous
- safe
- hazardous
- acceptable
- dependable
- acute
- no effect
- tolerable
- minimal
- worst-case
- best-case

### Comparative

Risks which Increase the Probability of Death. by One Chance in a Million (Crouch and Wilson, 1982):

> Compare the numerical risk to other low-probability events that are more familiar

- Smoking 1.4 cigarettes a day
- Traveling 10 miles by bicycle

- Traveling 300 miles by car
- One chest x-ray taken in a good hospital
- Eating 40 tablespoons of peanut butter

### Numerical Measures

> The technical description of risk uses the language of probability and statistics.

- Probability
  - = 10 percent chance of rain
  - = 1 in 10 chance of rain
  - = probability of .1 of rain
  - = 9 to 1 odds against rain
- Statistics
  - = the average number of incidents is .25 per year
  - = the standard deviation is .15
  - = 2.5 additional deaths per 1,000 people
  - = premature loss of 1.5 years of life
- Distributions
  - = more complete definitions of probabilities at different levels
  - = can be described mathematically (by a function) or by a curve defined by points

## RISK-RISK

Reducing risk in one area can lead to increases in risk in other areas, or at other times. An examination of the interactions associated with risk reduction is called risk-risk analysis. A full accounting of risks associated with a proposal should include a description of all increased and decreased risks associated with that proposal.

Examples of risk reductions that can change risk in other arenas:

- Levee Construction
  - = protect certain areas
  - = induce higher flooding, higher risks in other areas, greater consequences under failure due to higher head and greater development behind the levee
- Chlorination of water
  - = reduces risk of waterborne disease
  - = increases amount of trace carcinogens in drinking water
- Nuclear Power
  - = reduces dependence on other energy sources
  - = increases risk of population exposure to radiation
- Antibiotics in Animal Feed
  - = reduced disease in animals, growth promoters, appetite enhancers for animals

- = increase possibility of development of resistant strains of bacteria
- Missile Defense
  - = may reduce risk of terrorist attack
  - = possibly destabilizing, increasing risk of attacker's first strike
- Vehicle Fuel Efficiency
  - = reduce health risks associated with air pollution
  - = smaller cars increase automotive fatalities

## ATTITUDES TOWARDS RISK

Individuals (and groups) vary significantly in their attitudes and preferences towards risk, which exist on a continuum. General terms/concepts used are:

- risk averse—tendency to avoid gambles
- risk neutral—indifferent to equivalent outcomes in a gamble
- risk seeking—preference for a gamble

"The concept of risk aversion is linked with the idea of a fair bet. A fair bet is an uncertain prospect whose expected yield is zero. A person is risk averse if he never accepts a fair bet. A person is called a risk lover if he always accepts a fair bet. If a person is always indifferent between accepting a fair bet and rejecting it, he is called a risk neutral person. Note, one may not belong to any of these three categories. Consider the following tow cases. A coin is tossed. In the first case, if head comes up the gambler receivers $1. If tail comes up then he will have to pay $1. This is a fair bet with a zero expected return. In the second case, the sum is raised to $1000. It is not unusual for a person to play the first bet and refuse to play the second bet. He is a risk lover for small bets and a risk averse person for large bets " (Biswas, 1997)

There is little consistency in how people deal with risk in their own lives. Someone who is risk averse in one arena (financially conservative) may be risk-seeking in another (sky-diving). Studies show contradictions in people's responses, but there are some general principles:

- There is a preference for avoiding loss
- A sure gain is preferable to a possible larger gain
- A gamble is preferable to a sure loss

Externally imposed risks (nuclear power plants) are less acceptable to individuals than voluntarily assumed risks (smoking, skiing).

Statistical risk is viewed differently from individually identifiable risk:

"If an individual or a group can be identified as the bearer of risk as opposed to statistical populations at risk, society tends to value this identifiable risk with increased concern, assuming equal probabilities and identical consequences, over statistical risk. For example. [prior to the Coal Mine Health and Safety Act, 1969, 1973] ... many mine companies made little investment to protect miners as a group; but whenever a mine disaster occurs, millions are spent to rescue trapped miners, dead or alive.

If the risk taker expects to experience a consequence directly as an individual, he generally attributes a higher value to the consequence (positive or negative) than if he is only one of a group of people for which only one or a small number will experience the consequence." (Rowe, 1977)

## RISK AND TIME

Decision problems where the key results are both risky and vary over time are pervasive. Virtually all capital investments, budgeting problems, or planning situations possess time and risk dimensions in their evaluation. Money must be spent in the short term to reduce long-term risks.

Uncertainty about specific future events is greater the longer the time horizon, but there is more certainty about average behavior over a long period of time.

Many choices present the issue of short-term satisfaction vs. long-term risks (e.g., global warming). The issue is often framed as that of deferring present costly action until more certain information is obtained, but deferring is in itself a choice. Future risks tend to be discounted.

In many Corps projects, the concept of life-cycle modeling is used in analysis. In life-cycle modeling, behavior over a defined period (50 or 100 years) is examined, and economic discounting is used to allow decisions to be based on present values. Under such assumptions, economic losses far out in the life cycle are weighed less than those that might take place in the near term. At the same time, other parameters, such as the population at risk from dam failure and ecosystem restoration outputs and benefits, are not discounted.

## REDUCING EXPOSURE TO RISK

Once risk is recognized as a fundamental characteristic of any possible action, there are a number of basic ways to reduce exposure to risk (Keeney, 1999):

Share the risk—other individuals or organizations can participate (e.g., a large group of investors);

- Seek risk-reducing information—obtain additional facts or insights that allow for risk-avoiding choices
- Diversify the risk—use a 'portfolio' of projects or investments ("don't put all your eggs in one basket")
- Hedge the risk—utilize various financial instruments (such as futures contracts for commodities) to 'lock in' a price
- Insure against the risk—purchase insurance against the worst outcomes

“All of these techniques help to manage risk by enlisting others in transactions that reshape the original risk profile, making it more compatible with the decision maker’s risk tolerance.” (Hammond, Keeney, Raiffa)

Note that hedging and insurance imply that others have a different opinion of the expected value of the outcome. In essence, an insurer is betting against you.

## RISK MANAGEMENT IN THE FINANCIAL WORLD

Other organizations and individuals use a variety of means to deal with risk. The volatility of the financial environment is much greater than it has been at many times in the past. This increased volatility has led, in the financial world, to the development of a variety of new risk management instruments and approaches (and, in some cases, to spectacular financial losses). Through a variety of techniques (e.g., options, swaps, forward contracts, and futures), and through sophisticated analyses of exposure, financial risk management has become an important feature in most major commercial enterprises. Two important technical developments, some 20 years apart, have been profoundly influential in the development of risk management techniques in the financial world.

In an article entitled “Portfolio Selection,” Markowitz (1952) addressed the problem of reducing the variance in the return of a stock portfolio. Apparently, prior to that time, the concept of risk was little addressed in the world of equity investments. [cf. Bernstein, 1998]. Markowitz’s contribution was significant in that it showed that portfolios could be assembled to have different mean return and variation of return (taken as risk). The higher the expected return, the greater the risk. Markowitz started the field of modern portfolio management theory, with an emphasis on diversification, risk, and return.

“Portfolio theory starts from the premise that investors choose between portfolios on the basis of their expected return, on the one hand, and the standard deviation (or variance) of their return, on the other. The standard deviation of the return can be regarded as a measure of the portfolio’s risk. Other things being equal, an investor wants a portfolio whose return has a high expected value and a low standard deviation. ... An investor who is very averse to risk will therefore choose a safe portfolio with a low standard deviation and a low expected return, and an investor who is less risk averse will choose a more risky portfolio with a higher expected return.” (Dowd, 1998)

A technique for pricing options to buy and sell stock was described in 1973 by Black and Scholes (Black, 1973). Prior to that time, there was no generally accepted method of determining a rational price for these financial instruments. An option is a right (but not the obligation) to buy or sell something at a designated price at a designated time in the future. The question of what is a fair price to pay for an option is complex.

The so-called Black-Scholes Option Pricing Model started the development of a whole range of financial risk management vehicles. The Black-Scholes model looks at volatility of a stock and interest rates to determine the appropriate price of an option (to buy or sell the stock in the future) in the present. By allowing for rational pricing of options, the Black-Scholes and similar models allow for the construction of a variety of mechanisms for ameliorating various types of financial risk through the construction of offsetting transactions. Thus, for a known price, it is possible to place a floor on exposure to such things as future interest rate or foreign exchange rate changes.

The portfolio management approach leads to a holistic view, rather than an atomistic view, of risk and return. The options pricing model and its successors lead to the concept of active management of financial risk, through the combination of individual transactions to minimize total risk. Thus, in both cases, the important point is the move from examination of a single investment, to the concept that multiple investments are essential to the management of risk.

Among the important characteristics of the modern commercial approach to risk management are:

- it is enterprise-wide—each project is examined as part of the overall enterprise, each stock is examined as part of the portfolio
- a variety of techniques are used to manage risk exposure
- it relies upon extensive historical data about past investments and performance

The sophisticated use of risk management techniques in the financial world relies upon the existence of a liquid market for a variety of financial instruments. As well, a single, unitary measure (money) is used. Risk management techniques in the financial world have been extensively studied, in part because of the financial incentive for development of good techniques, and the great losses that can be (and have been) incurred by poor risk management approaches.

Application of these techniques in the Corps is limited by a number of circumstances:

- each Corps project is viewed individually, not as part of a 'portfolio' of projects
- there is only a limited market for instruments (such as flood insurance) to offset risks associated with the Corps projects, and these instruments are not necessarily priced appropriately from an economic or policy perspective
- guidance dictates that projects be evaluated based primarily on expected value of net NED benefits; where risk assessment takes place, the impact on the decision process is primarily through the determination of expected value rather that through the variance
- there is little historical data on economic performance of projects, to allow for an ex post facto examination of the validity of risk assumptions

While the Corps cannot directly create markets for risk management instruments, steps can be taken to allow for more flexible risk management approaches:

- view certain types of projects (e.g., environmental restoration at multiple sites) from a 'portfolio' perspective, that is, as a group, rather than individually;
- encourage rational pricing of insurance alternatives to the Corps physical measures;

- compile data bases on project economic performance as compared to the economic justifications used for the projects.

## *APPENDIX B*

## *RISK IN THE CORPS PLANNING PROCESS*

# APPENDIX B: RISK IN THE CORPS PLANNING PROCESS

The Corps guidance specifies that risk and uncertainty will be incorporated in the water resource planning process:

"The Principals and Guidelines state that planners shall characterize, to the extent possible, the different degrees of risk and uncertainty inherent in water resources planning and to describe them clearly so decisions can be based on the best available information.

Risk-based analysis is defined as an approach to evaluation and decision making that explicitly, and to the extent practical, analytically incorporates considerations of risk and uncertainty.

Risk-based analysis shall be used to compare plans in terms of the likelihood and variability of their physical performance, economic success and residual risks.

A risk-based approach to water resources planning captures and quantifies the extent of risk and uncertainty in the various planning and design components of an investment project. The total effect of risk and uncertainty on the project's design and viability can be examined and conscious decisions made reflecting an explicit trade-off between risk and costs." (ER 1105-2-100, April 2000)

The Corps planning process comprises the following steps:

- Identifying Problems and Opportunities
- Inventorying and Forecasting Conditions
- Formulating Alternative Plans
- Evaluating Alternative Plans
- Comparing Alternative Plans
- Selecting a Plan

Uncertainty is clearly present in forecasts, which carries through all subsequent steps. The recognition of uncertainty in estimates of costs, benefits, and impacts falls under the evaluation step, and is the arena in which risk assessment is most prevalent within the Corps. Comparison of alternative plans has generally been done based on expected value, without extensive display of the risk-related issues. The selected plan is frequently (but not always) the plan with the highest expected value.

Opportunities exist to enhance the communication and display of risk characteristics in plan comparison, and to use improved decision analysis techniques taking into account risk in plan selection.

## CORPS APPLICATIONS: CORPS BUSINESS PROCESSES

The work of the Corps is organized within nine business processes:

- Navigation
- Flood Control / Coastal Protection
- Environmental Impact / Ecosystem Restoration
- Hydropower
- Water Supply
- Recreation
- Emergency Management
- Regulatory Functions

Risk and uncertainty play roles in each of these arenas, with varying levels of application of tools, techniques, and models to date. Specific applications of the risk-based approach are discussed for selected Civil Works areas.

## Navigation

Navigation projects encompass ports, harbors, channels, and waterways. Both inland and coastal activities are undertaken.

Appendix E of ER1105-2-100 notes that risk analysis procedures for inland navigation are under development:

"Although these efforts are ongoing, preliminary indications are the following variables should be explicitly incorporated in risk-based analysis; 1) commodity forecasts, 2) alternative mode costs, 3) reliability of existing and proposed structures, and, 4) system delays associated with capacity constraints. Additional variables can be incorporated if appropriate for individual study areas. Districts are expected to incorporate risk-based analysis procedures in all inland navigation studies. Until risk-based procedures are fully developed, districts are expected to, at a minimum, perform sensitivity analysis of key variables."

Further:

"Project benefits are calculated on the basis of "the most probable" with project and without project conditions. However, risk and uncertainty should be addressed in the analysis of NED benefits and costs. In particular, major uncertainty exists in the proper measure of savings to shippers, namely the difference in long-run marginal costs. To the extent that rates or other prices vary from long-run marginal costs, savings to shippers will contain a component of transfers varying from real resource savings. This element of uncertainty should always be identified or acknowledged in estimates of benefits."

"The Institute of Water Resources is currently developing risk-based analysis procedures for deep-draft navigation studies. Unlike the current risk-based flood damage model, the navigation model will integrate both benefit uncertainty, related to fleet and commodity forecasts and vessel operating costs, with cost uncertainty related to dredging and disposal costs. Districts are expected to continue to use risk and uncertainty techniques in all navigation studies, at least in the form of sensitivity analyses, before field release of the risk-based navigation models."

Uncertainty is present in many aspects of navigation analysis. There is uncertainty in commodity forecasts that determine planning for ports and harbors. Required amounts of dredging are highly variable in space, time, and quantity. As noted above, alternate mode transportation costs are uncertain. Reliability of navigation structures is dependent upon aging and failure of various components, some of which may be damaged through accident. Navigation improvements are frequently justified based on reductions in transit times for commercial traffic, which are dependent on a variety of probabilistic processes (arrival and servicing of tows at locks, collisions, water levels).

The Institute for Water Resources (IWR) has developed three models that use Monte Carlo simulation techniques to include uncertainty in assessments of navigation improvements and major rehabilitation to locks. The Navigation-Repair model was designed for use in major rehabilitation studies of navigation locks. It simulates the life cycle of a single navigation lock as tows pass through. The lock is composed of components (e.g., gates and valves) that can assume different states, to represent various levels of performance. Probabilities of state transitions are defined and associated with events (such as a lock cycle, or passing of time). Lock service times are dependent upon the state of the components. The model can be used to examine alternative major rehabilitation strategies for component repair/replacement. The NavSym model is a system-wide Monte Carlo simulation model for waterways (including locks), that has been applied by the Galveston and Mobile Districts for navigation improvement studies. The model determines overall savings in transit time based on alternative navigation improvements. The HarborSym model is used for examination of improvements to ports/harbors used by deep draft vessels, and has been used by the Galveston District in an examination of improvements to the Sabine-Neches area.

## Flood Control/Coastal Protection

Hydrologic processes have been extensively studied as probabilistic systems. Flood damage analysis is probably the first and most highly evolved risk assessment used within the Corps. Risk analysis is specified in the guidance:

"Flood damage reduction studies are conducted using a risk-based analytical framework. The risk framework captures and quantifies the extent of the risk and uncertainty and enables quantified tradeoffs between risk and cost. Decision making considers explicitly what is gained and what is lost." (ER1105-2-100). Additional information is provided in ER 1105-2-101 and EM 1110-2-1619 .

"Flood damage reduction studies are conducted using a risk-based analytical framework. Models, data, and measurement and many physical, social, economic and environmental conditions are subject to variation and uncertainty. This has been long known, if in the past incompletely acknowledged. Management by routine overbuilding and freeboard are not affordable. The risk framework captures and quantifies the extent of the risk and uncertainty, and enables quantified tradeoffs between risk and cost. Decision making considers explicitly what is gained at what cost." (ER1105-2-100, Appendix E)

Flood damage analysis is based on a variety of probabilistic processes – probability of a particular streamflow (flood frequency), which is then related to probability of the streamflow being translated into a particular stream stage (stage-discharge), and which leads to the probability of damage to structures based on the stream stage (stage-damage). The Hydrologic Engineering Center (HEC) has developed the HEC-FDA analysis tool to assist in this process.

Similar approaches are present in guidance for storm damage reduction: "Storm damage reduction studies should adopt a life cycle approach and probabilistic analysis (and display) of benefits and costs. Key considerations are listed below; at a minimum, those with the greatest effect on plan formulation should be explicitly incorporated in the analysis.

a. The erosion damage function (with special emphasis on structure values and land values)
b. The stage-damage function (with special emphasis on structure first floor elevation, content and structure values
c. The wave-damage function by structure class
d. Storm-related parameters such as peak wave height and period storm duration, peak surge elevation, and timing with respect to tidal phasing
e. Wave height above the dune
f. Wave penetration
g. The shoreline retreat or eroded volume
h. The natural post-storm recovery" (ER1105-2-100, Appendix E)

Risk-based tools for shore protection include the GRANDUC (Generalized Risk AND Uncertainty) model developed by the Wilmington District, and the Storm Damage Model (SDM) developed by the Jacksonville District. Both models make use of a structure inventory and incorporate structural damages based on probabilistic events relating to storms. As of this writing (September 2002), an effort is under way to prepare the design of a unified, life-cycle, event-driven Monte Carlo simulation model for shore protection projects, that will incorporate risk/uncertainty in storm events, beach response, and structural damages due to flooding, waves, and erosion.

The Hoover Dike model, developed by IWR for the Jacksonville District, is an economic life cycle Monte Carlo simulation of levee protection that was used in the assessment of a variety of major rehabilitation proposals for the Hoover Dike surrounding Lake Okeechobee. While the economic analysis did not show a benefit-cost ratio greater than 1.0, the project was justified in part by demonstrating the uncertainty in consequences, as shown in the accompanying Figure B-1 (taken from the Herbert Hoover Dike Major Rehabilitation Evaluation Report, Appendix B, 2000) which gives the cumulative frequency distribution of the benefit-cost ratio, as developed from the Monte Carlo simulation model.

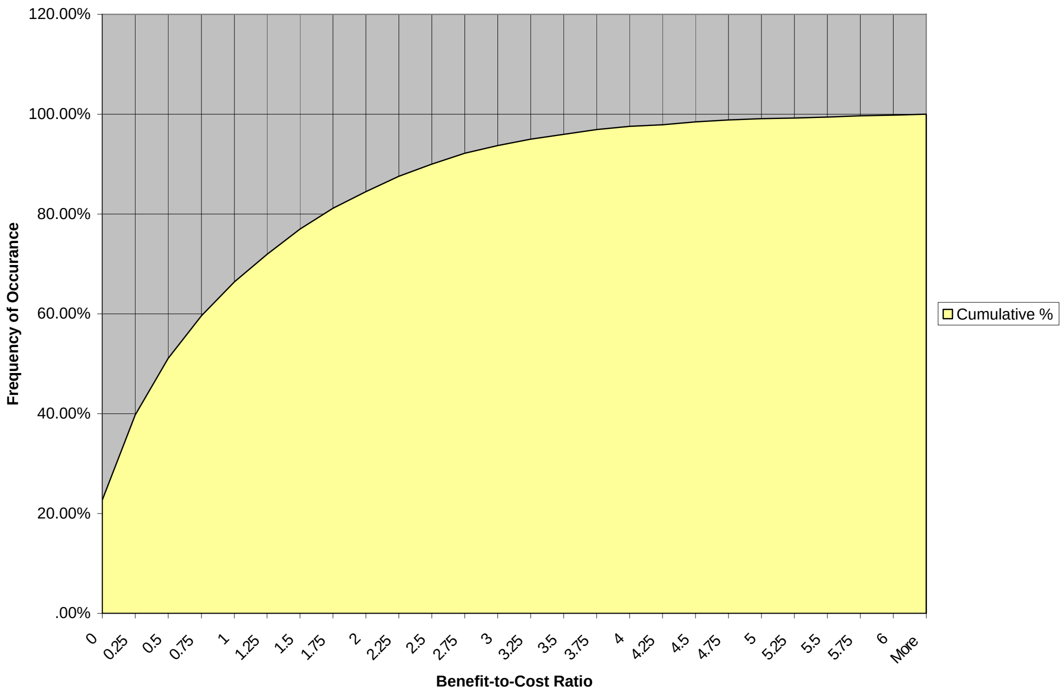


**FIGURE B-1**
**CUMULATIVE FREQUENCY HISTOGRAM**

## Environmental Impact/Ecosystem Restoration

Guidance (Appendix E of ER 1105) specifies the incorporation of risk and uncertainty in ecosystem restoration:

"Risk and Uncertainty Considerations. When the costs and outputs of alternative restoration plans are uncertain and/or there are substantive risks that outcomes will not be achieved, which may often be the case, the selection of a recommended alternative becomes more complex. It is essential to document the assumptions made and uncertainties encountered during the course of planning analyses. Restoration of some types of ecosystems may have relatively low risk. For example, removal of drainage tiles to restore hydrology to a wetland area. Other activities may have higher associated risks such as restoration of coastal marsh in a area subject to hurricanes. When identifying the NER plan the associated risk and uncertainty of achieving the proposed level of outputs must be considered. For example, if two plans have similar outputs but one plan costs slightly more, according to cost effectiveness guidelines; the more expensive plan would be dropped from further consideration. However, it might be possible that, due to uncertainties beyond the control or knowledge of the planning team, the slightly more expensive plan will actually produce greater ecological output than originally estimated, in effect qualifying it as a cost effective plan. But without taking into account the uncertainty inherent in the estimate of outputs, that plan would have been excluded from further consideration."

There is a good deal of knowledge uncertainty in the models (Habitat Evaluation Procedures [HEP]) used for much of the analysis of ecosystem restoration. An extensive discussion of the issues and techniques associated with ecosystem restoration costs is contained in Yoe (2002).

There is a requirement in the guidance for cost-effectiveness and incremental cost analysis. The IWR-PLAN tool is one such method for carrying out these analyses for a variety of different types of projects. At present (2002), there are plans to incorporate risk assessment capabilities within IWR-PLAN.

## Hydropower

Hydroelectric power generation has natural uncertainties in flow, demand and performance of physical facilities. The Corps activities in hydropower include development and operation of hydropower facilities, and major rehabilitation of existing facilities. There is no requirement in the guidance (ER1105-2-100) for risk and uncertainty analysis in determining NED benefits of new hydropower development, but the Corps involvement in such projects at present is limited. The majority of the Corps participation in hydropower projects involves major rehabilitation at existing facilities. Major rehabilitation studies examine the probability of unsatisfactory performance of project features, with attendant reduction in project outputs. Guidance for major rehabilitation of hydropower facilities specifically recognizes risk and uncertainty in unsatisfactory feature performance:

"Considerable risk and uncertainty is inherent in the base condition. The timing, frequency, and consequences of system disruption are all unknown and must be estimated. The analysis should explicitly show the effects of reasonable alternative assumptions concerning these variables." (ER1105-2-100, Appendix E, Section X)

As well, the guidance specifies a number of steps that can be carried out, suggesting the use of Event Trees and Monte Carlo simulations, noting that: "An advantage of the Monte Carlo approach is that it yields both the expected value and the variance."

Monte Carlo simulation techniques have been applied to major rehabilitation studies within the Corps. The Portland District has used models for studies of several hydropower rehabilitation proposals, including the Dalles. IWR has developed a general-purpose tool, HydroPower-REPAIR, which has been used by the Omaha, Savannah and Mobile Districts, and by the Hydropower Analysis Section of Northwestern Division. HydroPower-Repair analyzes the economic behavior of hydropower facilities composed of generating units and their components over a project life cycle. Components may fail, leading to outages, downtime and costs for repairs. The tool allows for analysis of tradeoffs between emergency repair and rehabilitation.

## Water Supply (Municipal and Irrigation)

Guidance for M&I Water Supply NED Benefits evaluation requires risk analysis:

"The procedures presented apply to both structural and nonstructural elements of such plans. Risk-analysis techniques are required in all formulation, evaluation and investment decision studies. No specific risk-based procedures have been developed for municipal and industrial water supply analysis. For studies and projects where water supply benefits constitute a substantial portion of total benefits, analysts are expected to perform, at a minimum, sensitivity analysis of key variables such as least cost alternative cost, future demand for water and future availability of water supplies." (ER1105-2-100, Appendix E, Section VII, E-55)

# *APPENDIX C*

# *THE QUANTITATIVE DESCRIPTION OF RISK*

# APPENDIX C: THE QUANTITATIVE DESCRIPTION OF RISK

The quantitative description of risk requires the use of probability and statistics. Probability is a measure of the likelihood of something happening, and statistics are numerical measures that summarize and describe larger amounts of information.

The probability of a particular event is described numerically as a value between 0 and 1, with 0 meaning the event is impossible, and 1 meaning that it is certain. This is referred to as a point estimate of probability, and can be used to describe, for example, the chance that a coin toss will result in heads or tails, or the probability of throwing a two with a pair of dice.

When a range of possibilities exists for an outcome, then the combination of point estimates of probabilities is expressed as either a discrete or continuous curve known as a probability distribution. Certain shapes of distributions occur frequently and are useful in many circumstances, because they describe many phenomena quite well. The Gaussian or Normal Distribution (the bell-shaped curve) is an example.

Often, shorthand measures of distributions are used to characterize distributions with a few numbers. Typical measures are the mean and the standard deviation. Such measures are called the parameters of the distribution. Confidence limits describe a range of certainty about estimates, e.g., 95 percent certainty that the mean of a distribution lies between the values of 3 and 9. The larger the confidence limit range at a level (e.g., 95 percent), the less certainty there is about an estimate.

Probability is used within the risk assessment phase of risk analysis by defining, for the important variables that are taken as uncertain, the probability distribution associated with the values that the variable can take. This can be done by explicitly defining the distribution (e.g., giving the coordinates of the points on the distribution curve), or by selecting a generic type of distribution (e.g., normal, log-normal, triangular, etc.) and the parameters of that distribution, leading to a mathematical definition of the curve. These probabilities are most often set in one of two manners: through statistical analysis of historical data, or through expert opinion.

## PROBABILITY

Probability methods of estimation are referred to depending upon how the probability is obtained:

- objective / empirical / frequentist probability
  - = Can be measured (based on sample statistics of a population): how many times does the event of interest happen out of the number of times it could have happened?
  - = When there is no relevant population, e.g., a population not yet exposed to a new risk, it is not possible to obtain an objective probability.

“The classical or frequentist view of probability defines the probability of an event’s occurring in a particular trial as the *frequency* with which it occurs in a long sequence of similar trials. More precisely, the probability is the value to which the long-run frequency converges as the number of trials increases.

The problem is that for most events of interest for real-world decision making, it is not clear what the relevant population of trials of similar events should be.” (Morgan and Henrion, 1990)

- Analytical probability
  - = Based on physical / theoretical / mathematical constructs
  - = Chance of heads with a fair coin = .5: based on assumptions about physical properties of the coin
  - = Chance of rolling a total of 7 with a pair of fair dice = 1/6: based on assumptions about physical properties of the dice, plus mathematical combination of the ways that 7 can be thrown (6+1,1+6,5+2,2+5,4+3,3+4) = 6 out of a total of 36 possible combinations for two dice
  - = Assumes certainty about the model used to derive the probability (e.g., the coin is fair)
- Bayesian / Personalist / Subjective probability
  - = requires personal estimates
  - = evidence / experience based
  - = expert opinion
  - = what happens if experts disagree?

“A probability of an event is the degree of belief that a person has that it will occur, given all the relevant information currently known to that person. Thus, the probability is a function not only of the event, but of the state of information.

Since different people may have different information relevant to an event, and the same person may acquire new information as time progresses, there is strictly no such thing as ‘the’ probability of an event. Different people or one person at different times may legitimately assign different probabilities to the same event. [(organ and Henrion, 1990)

It is often preferable to work with empirical or analytical probabilities, but this is not always possible. Even with empirical/analytical probabilities, there are always questions about the population being examined, e.g., has a watershed changed such that historical streamflow information is no longer representative of how the flows will behave in the future.

## PROBABILITY DISTRIBUTIONS

In order to describe the probabilities associated with a phenomenon with many possible outcomes, a probability distribution is used. If the range of outcomes is limited to a defined set of possibilities (i.e. the total number thrown with two dice, limited to 2 through 12), then a discrete probability distribution can be used, as displayed in Figure C-1, showing the probability of each outcome. The ascending cumulative probability shows the probability of obtaining the given value or less. When all possible outcomes are specified, then the maximum cumulative value must be equal to one.  An example of this is shown in Figure C-2.

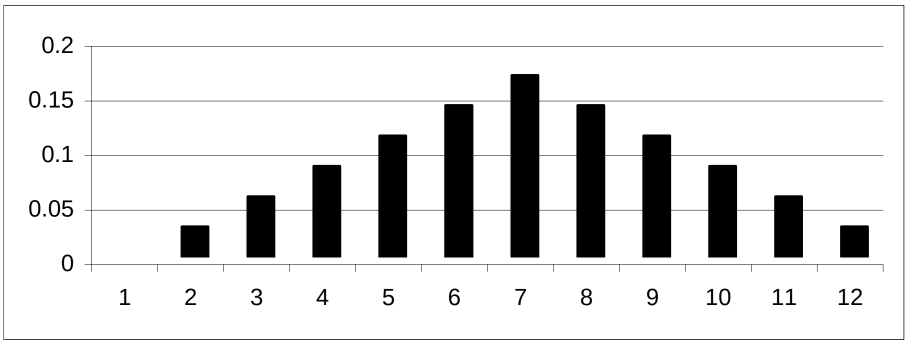


**FIGURE C-1**
**DISCRETE PROBABILITY (TOTAL OF TWO DIE)**

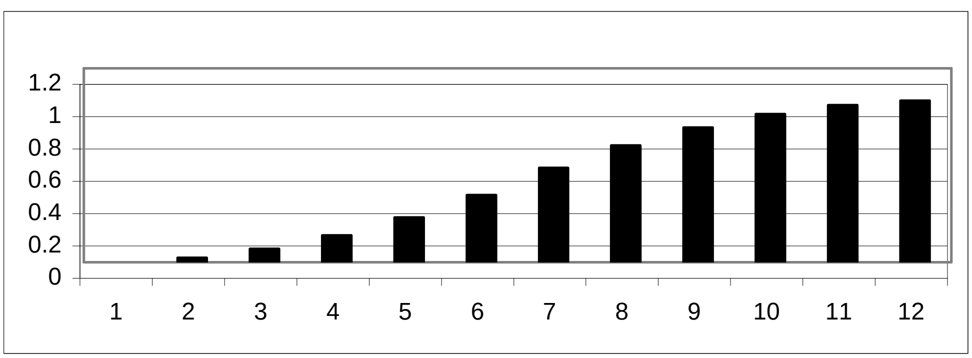


**FIGURE C-2**
**CUMULATIVE PROBABILITY**

## Probability Density Function (pdf)

When the outcomes are best described as a continuous rather than a discrete variable, then a density function is used to describe the distribution, as a continuous curve. The graph in Figure C-3 shows the distributions associated with a normal distribution with mean of 5.0, and a standard deviation of 2.0.

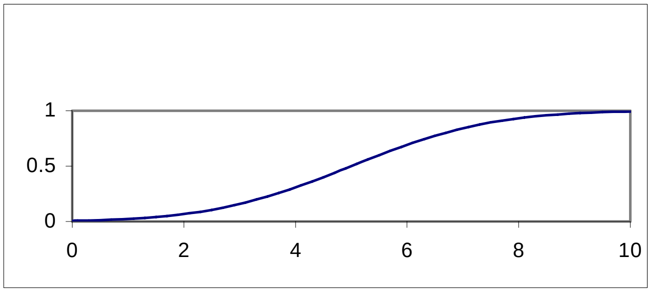


**FIGURE C-3**
**CUMULATIVE DENSITY FUNCTION**

The probability density function provides a good display of:

- the relative probabilities of different values.
- the mode(s) [most likely values] as peak(s) in the curve
- the general shape of the distribution (symmetry, etc.)
- small changes in probability density

The area under the probability density function is equal to 1.0.

## Cumulative Density Function (cdf)

The cumulative distribution is usually displayed as showing the probability of obtaining a value less than or equal to a given value (ascending CDF). With a normal distribution with mean of 5 and standard deviation of 2, the probability of obtaining a value less than or equal to 7.5 is 0.894. The cumulative distribution always runs from 0 to 1. The steeper the slope of the cdf, the less variability there is in the distribution. The cumulative function best displays probability of intervals, and other measures such as the median ( the value at which the probability is .5).

The pdf and cdf (discrete or continuous) provide good ways of visualizing the probability associated with a range of outcomes. In general, it is best to display both curves, as each shows different features associated with the situation.

“Probability distribution functions arise from the fundamental properties of the quantities we are attempting to represent. Quantities formed from adding many uncertain quantities tend to be normally distributed, and quantities formed from multiplying uncertain quantities tend to be lognormal. Events that occur randomly in time lead to exponential and Poisson distributions.” (Morgan and Henrion 1990)

## PARAMETERIZED DISTRIBUTIONS

In many cases, distributions can be described mathematically as functions of a small number of parameters. There are a number of well-known distributions of this form. When ‘real-world’ data can be described by a distribution described by parameters, the probabilities can be represented in a short-hand form based on the parameters. For example, a gaussian or normal distribution is described by two parameters, the mean and standard deviation. As well, other quantities of importance can be developed from the known parameters and the mathematical representation of the distribution.

A number of mathematical formulations of distributions have proven important in representing real-world processes. Among these are:

- Normal (Gaussian) – describes variation around some central value, such as measured height or weight, and the typical distribution of measurement error – two parameters, mean and standard deviation
- LogNormal – the log normal distribution describes a situation where the logarithm of a variable is normally distributed. – two parameters, mean and standard deviation of underlying normal distribution

It is “good for physical quantities that are constrained to being non-negative and are positively skewed, such as pollutant concentrations, stream flows, or accident event magnitudes (e.g., spill quantity or explosion intensity). The lognormal distribution is particularly appropriate for representing large uncertainties that are expressed on a multiplicative or order-of-magnitude basis.” (Morgan and Henrion, 1990)

- Uniform – useful when a range of possible values is known, but there is no more information about the relative likelihood of the intermediate values, so all values in the range are assumed to be equally probable – two parameters, maximum and minimum
- Poisson – describes the number of events that occur in a fixed time period when the average rate of events per unit time is known – one parameter (average rate of events/time)
- Exponential – describes the time between successive events, such as accidents (accidents, storm event durations, spill sizes) – single parameter distribution
- Triangular – useful when describing a phenomenon that has a limited range, and a most likely value – described by three parameters – minimum, most likely, and maximum – frequently good for quantifying expert opinion

These, and other parameterized distributions (Gamma, Weibull, etc.) are useful in constructing models of complex systems. Choosing an appropriate distribution to describe a particular variable, and assigning parameters to that distribution, is a basic part of such model-based risk assessment techniques as Monte Carlo simulation.

## EXPECTED VALUE

Expected value is a fundamental concept of probability and statistics that is used in decision-making under risk and uncertainty. The expected value of a variable is the arithmetic mean or average over all possible outcomes, when samples are repeated. For parameterized distributions, the expected value can be developed mathematically from the parameters of the distribution.

Expected value is also used in the sense of 'expected payoff', for a gamble or lottery, for example. In that sense, it is the sum of the probability of a given outcome multiplied by the value of that outcome, over all the possible outcomes (assuming independent trials and a large number of trials). In a roulette game (on a wheel with 36 numbers and 0 and 00), the normal return for a $1 bet on a single number is $36.00 (35 to 1 payoff), while the odds of achieving that number are 1/38. Thus, the expected value of a $1 bet is the sum of the probabilities:

(1/38 x $36 return) + (37/38 x $0 return) = $ 0.94736

This expected value only holds true over a long series of trials.

Expected value is used as a primary decision criterion – if the expected value of the result is greater than the investment, then the investment is a 'good' one, and if the expected value of the result is less than the investment, the investment is not a good one. The existence of gambling and insurance, where the expected value is always in favor of the insurer or the casino, proves that expected value is not the only decision criterion that is used by an individual in assessing an investment.

In 1738, Daniel Bernouilli published a paper "Exposition of a New Theory on the Measurement of Risk" on the so-called St. Petersburg Paradox, pointing out a severe problem with the use of expected value as a decision criterion. [cf. Bernstein, 1998, for a description]. The St. Petersburg Paradox describes a proposed gambling game, with an infinite expected value. A coin is flipped until it comes up tails, and the total number of flips, n, determines the prize, which equals $\$2^n$. Thus if the coin comes up tails the first time, the prize is $\$2^1$ = $2, and the game ends. If the coin comes up heads the first time, it is flipped again. If it comes up tails the second time, the prize is $\$2^2$, = $4, and the game ends. If it comes up heads the second time, it is flipped again. And so on. There are an infinite number of possible 'consequences' (runs of heads followed by one tail) possible. The expected value of this game, when calculated mathematically, is infinite: The 'expected value' of the game is the sum of the expected payoffs of all the consequences. Since the expected payoff of each possible consequence is $1, and there are an infinite number of them, this sum is an infinite number of dollars. The question associated with the St. Petersburg Paradox is: "How much would you pay to play this game?" According to expected value theory, the payoff is infinite, but most people would not pay a very large sum to play, recognizing that the chance of getting a run of heads long enough to justify the investment is unlikely.

Various explanations have been offered for the difference between clear mathematical theory and rational behavior. Obviously, the budget of the game player is limited – there is not infinite time, nor infinite money, to play the game. The St. Petersburg Paradox demonstrated that expected value, by itself, is not in general a sufficient criterion for decision-making. Expected monetary values do not always accurately reflect choices made in practice– this is particularly true when losses or gains involved are large compared with the resources available.

As another example of a situation where expected value is not necessarily a good decision criterion, consider a situation in which the owner of a small business is deciding whether or not to undertake one of two contracts, contract A and contract B (example from Moore, 1972). There is uncertainty about the outcome of each contract. The outcomes and probabilities are shown in Table C-1.

**TABLE C-1**
**OUTCOMES AND PROBABILITIES**

| Outcome | Contract A | | Contract B | |
|---|---|---|---|---|
| | Payoff | Probability | Payoff | Probability |
| 1 | 80,000 | 0.6 | 50,000 | 0.5 |
| 2 | 10,000 | 0.1 | 30,000 | 0.3 |
| 3 | -30,000 | 0.3 | -10,000 | 0.2 |

Under an expected value analysis (summing probability x payoff for each outcome), contract A yields an expected value of 40,000, while contract B has an expected value of 32,000. Based on the expected value calculation, the business owner should accept contract A. “But it is by no means certain that all businessmen who found themselves in this situation would choose A rather than B. The reason is that under A there is some possibility of a loss of 30,000, and such a loss might wipe out the business completely. Hence, they might not be willing to undertake any contract which held out some possibility of so large a loss, regardless of the fact that there was some possibility of gaining 80,000, and that the expected value of accepting the contract was as high as 40,000. If such a businessman had to choose between A and B he might well choose B because the maximum loss of –10,000 is much smaller, even though the expected value is nor rather lower. However, other businessmen for whom the loss of –30,000 would not be such a serious event, or who need desperately to obtain more than 50,000 to keep the business going, would probably prefer contract A.

## UTILITY AND EXPECTED UTILITY

The concept of utility can be used to explain the St. Petersburg Paradox, and decisions to play the lottery, gamble, and buy insurance, which appear to violate the criterion of selecting based on expected value. Utility is a measure of how valuable something is to someone – that is, it takes into account the preference and desire structure of an individual. The utility of $1 is obviously different to an extremely wealthy person and a penniless one. Thus, utility measures not only a thing itself, but how strongly the thing is desired.

For a lottery player, the utility of a possible large gain is greater than the utility of the certain small loss associated with buying the lottery ticket. For a purchaser of insurance, the avoidance of a large possible loss has greater utility than the certainty of the insurance payment.

Utility is a construct that has been developed to provide a structure to clearly observed behaviors, in particular the unequal valuing of monetary gains and losses at different levels. Typically, a utility function is defined from 0 to 1, with 0 being associated with the lowest utility, and 1 the highest utility. Values of the variable (e.g., monetary gain) are then mapped to the utility function. Consider the previous business example of two contracts with different payoffs, again drawing upon Moore (1972). Including the possibility of not choosing contract A or B (with an assumed value of 0), there are seven possible payoffs, in decreasing order:

80,000
50,000
30,000
10,000
0
-10,000
-30,000

By convention, the lowest value is assigned a utility of 0, the largest a value of 1, and various methods are used to determine the utility of the intermediate points. These methods typically involve determining the indifference of the decision-maker between probabilistic and certain outcomes, through what is called the lottery equivalent method. By asking a series of questions related to decision-maker preferences, it is assumed that the full numerical utility curve can be estimated. Assuming that such a process has been carried out, utilities are assigned as shown in Table C-2.

**TABLE C-2
ASSIGNED UTILITIES**

| Value | Utility |
|---|---|
| 80,000 | 1. |
| 50,000 | .9 |
| 30,000 | .8 |
| 10,000 | .5 |
| 0 | .3 |
| -10,000 | .2 |
| -30,000 | 0 |

Note that the utility function was developed independent of the probabilities of the payoffs. Graphically the curve is displayed in Figure C-4, showing a typical utility function in which there is an aversion to loss:

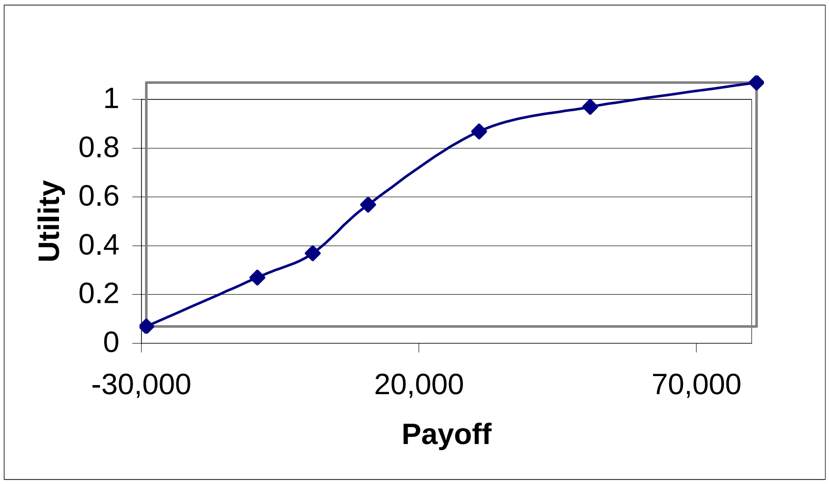


**FIGURE C-4
UTILITY FUNCTION**

The utility function can then be used to improve upon the expected value analysis, by replacing the monetary payoffs with utilities, and calculating the sum of (utility x probability) for each outcome of the two contracts. The resultant number is the *expected utility*. Under this method, contract A has an expected utility of 0.65, while contract B has a utility of .73, while not accepting either contract has an expected utility of 0.3. Of the three options, contract B is preferable under this analysis, which has included the preference for accepting a smaller expected profit to avoid some possibility of a large loss.

By going to the effort of creating the utility function, the risk associated with each alternative is, in theory, rationally included in the expected utility analysis. While this is an attractive approach to moving beyond expected value, there are a number of methodological problems associated with development of a utility function, particularly where there are multiple decision-makers.

## EXTREME VALUES – OPTIMISTIC AND PESSIMISTIC (BEST AND WORST CASE) SCENARIOS

Given the limitations associated with using expected value as the sole criterion for decision-making under uncertainty, and the problems of developing utility functions, other methods of displaying the range of possibilities and associated risks should be considered. As noted above, the display of a probability distribution is the most complete method, but other, simpler measures are frequently used, often derived from the distribution. For example, given a probability distribution of net benefits associated with a particular plan proposal, the expected value is readily determined as the value of .5 on the cumulative distribution function. As with the above business example, a pessimistic or risk-averse decision-maker may be interested in the maximum probable exposure to loss, or the *worst case scenario*. In contrast, an optimistic and risk-seeking decision-maker might wish to know the *best case scenario* of maximum probable gain. By selecting a level (e.g., 5 percent or 95 percent) of probability and then a worst and best-case magnitude at those probabilities, it is possible to get additional metrics of risk that can be used in the decision process, in addition to expected value. In order to avoid consideration of highly unlikely possibilities for best and worst case scenarios, it is advisable to base the definition of such scenarios on a defined and explicit probability value, e.g., the best case at 95 percent probability. When there are considerations of human life and health, the extreme value probability of the worst case is often set higher than when purely economic/commercial factors are involved.

# *APPENDIX D*

# *RISK VISUALIZATION AND COMMUNICATION*

# APPENDIX D: RISK VISUALIZATION AND COMMUNICATION

With the increase in public participation in major governmental decisions since the 1970s, issues related to methods of communicating risk have received a good deal of attention. Powell (1996) notes that it is “a relatively new scientific endeavor, dating back to Starr’s 1969 paper, which attempted to offer a scientific basis for thresholds of risk which would be accepted by the public.”

“In response to rising public concerns about health and environmental risks, government agencies have increasingly sought improved means for communicating risk information to individual citizens and public groups. Part of this increased interest in risk communication stems from current difficulties and frustrations. Government officials are often frustrated by what they perceive to be inaccurate public perceptions of risk and unrealistic demands by the public for risk reduction. Citizens are often equally frustrated by the government’s seeming disinterest in their concerns, unwillingness to take action, and reluctance or unwillingness to allow them to participate in decisions that intimately affect their lives.” (Covello, et al, 1989)

The problems of risk communication include:

- the need to describe the nature of unfamiliar low-probability extreme events
- the difficulty of conveying simultaneously magnitude and probability over a range of outcomes
- the differing technical backgrounds and viewpoints of analysts and audiences

Risk communication may involve either the location of a single risk measure against other comparable measures (such as comparing risk from different activities), or may involve demonstrating the uncertainty associated with a particular outcome (showing the range and probability). These are two different problems.

Visualization of risk is frequently attempted through use of graphics. A number of different techniques have been developed, with varying success. In general, more complex techniques and graphics fail to communicate well. Most people have great difficulty visualizing in more than two dimensions. Thus, simpler displays are to be preferred.

## PUBLIC PERCEPTIONS OF RISK

Understanding the way the public perceives risk is important to proper communication. Sandman (1989) discusses two important dimensions of public perception of risk – **hazard** and **outrage**. Hazard is defined as how many people are how likely to incur how much damage if we do X, and outrage is everything else that is relevant about a risk except how likely it is to be harmful. This view has come to be widely held as a good explanation of public views of risk.

> "Expert risk assessments ignore outrage and focus on hazard, but citizen risk assessments are more a product of outrage than of hazard. That is, we consistently underestimate the hazard of risks that are low-outrage and overestimate the hazard of risks that are high-outrage. We do this because we care about outrage." (Sandman, 1989)

Sandman provides the following components of outrage:

- Voluntary versus Involuntary
  People who voluntarily assume risks naturally tend to consider them acceptable and thus underestimate the hazard
- Familiar versus Exotic
  Familiar surroundings lead to underestimation of risk, as does familiarity with the risk itself
- Dreaded versus Not Dreaded
  Some outcomes are much more dreaded than others (e.g., cancer vs. driving accidents)
- Diffuse in Time and Space versus Focused in Time and Space (Potential for Catastrophe)
  Airline safety vs. auto safety – airline passengers die in larger groups
- Controlled by the Individual versus Controlled by the "System"
  Putting oneself in the hands of a corporation or the government provokes a different level of concern
- Fair versus Unfair
  Are the risks and benefits going to the same people?
- Morally Irrelevant versus Morally Relevant
  Some risks are viewed as inherently evil, independent of the seriousness of the hazard

> "Sandman notes that the public generally pays too little attention to the hazard side of risks, and experts usually completely ignore the outrage side. These are two very different starting points and not surprisingly, experts and consumers often rank the relative importance of various risks very differently. Scientists, in general, define risks in the language and procedures of science itself. They consider the nature of the harm that may occur, the probability that it will occur, and the number of people who may be affected. Most citizens, in contrast, seem less aware of probabilities and the size of a risk, and much more concerned with broader, qualitative attributes, such as whether the risk is voluntarily assumed, whether the risks and benefits are fairly distributed, whether the risk is controllable by the individual, whether a risk is necessary and unavoidable or whether there are safer alternatives, whether the risk is familiar or exotic, whether the risk is natural or technological in origin, and so forth. (Powell, 1994)

## AUDIENCE

It is well recognized that understanding of the audience is important in any form of communication. A distinction should be made between communications with the general public (or better yet, with the multiplicity of groups that comprise the "general public"), and internal technical communications.

Among recommendations for communicating to the public at large are:

- Use simple graphical material
- Provide opportunities for learning
- Put risks into perspective
- Relate on a personal level
- Understand qualitative concerns
- Recognize impacts of subtle changes in problem formulation
- Identify the specific target audience
- Generate involvement
- Avoid high threat campaigns
- Use multiple channels and media
- Use peer and social relationships
- Be inventive

> "Begin by identifying a specific target audience. Target your communication to that audience. You can speak differently to the other engineers on your study team than you can to the general public or to the Governor's representative. Avoid high threat campaigns. Threats are unpleasant and people tend to turn off to such messages. Maximally effective messages should be vivid, evoking lifelike images within the mind; come from a credible source; be clear, specific, concise and concrete; be clearly applicable to the person receiving it." (Yoe, Personal Communication, 2000)

“Diagrams intended to be useful in describing policy analyses will effectively convey information about models, their structure, associated uncertainties, and implications – only to the extent that the basic ideas on which they are based (influence networks, functional representation, parametric analysis, probability density, maximization, cost-benefit analysis, etc.) are concepts familiar to and understood by the users. This suggests that specific techniques for graphical communication are likely to be of varying utility when employed by different user populations.

The suitability of display designs depends much on the training and experience of the intended audience. Many audiences have little understanding of probabilistic information. Of course, this may slowly change as uncertainty analysis becomes more common. When knowledge is limited and rusty, a sequence of displays of related information of increasing sophistication and complexity may enable readers to grasp in successive steps displays they would not grasp if they were presented only in their final form.

**In general, it is probably unwise to design displays only for the least skilled in the audience; exposure to displays of varying sophistication will help viewers improve their skills in interpreting graphs, and so develop more sophisticated consumers**. (emphasis added)” (Morgan and Henrion, 1990)

## COMMUNICATION USING COMPARATIVE RISK ANALYSIS

Comparative risk analysis uses the risks associated with common experiences to create an understanding of where the risk being communicated falls along the spectrum of risk. It is commonly used for communication of health impacts, but can also be used to position flood risk and similar catastrophic events. Comparative risk analysis ranks risks for the seriousness of the threat they pose, and shows how current options compare to other, familiar choices such as the risk of driving a car or flying in an airplane. As well, comparative risk analysis can display anomalies in how we respond to high and low-threat risks. It may be problematic in that it can ignore the “outrage factor.”

“The numerical outputs of risk analyses are often difficult for decision makers to interpret. One promising approach for improving the ability of decision makers and others to interpret the results of risk analyses is to provide them with a means for placing estimated risks in perspective. If decision makers who review risk analyses have information on the level of risk associated with other more familiar risks, they possess a conceptual “ruler” that can help them to understand and interpret risk analysis results. For this reason, comparative risk information is an important component of a comprehensive methodology for communicating the results of risk analysis.” [Merkhofer, Lee. W., The Use of Risk Comparison to Aid the Communication and Interpretation of the Risk Analyses for Regulatory Decision-Making]

| TABLE D-1<br>Loss of Life Expectancy (Days) Due to Various Causes | |
|---|---|
| Being unmarried & male | 3500 |
| Cigarette smoking & male | 2250 |
| Heart disease | 2100 |
| Being unmarried & female | 1600 |
| Being 30% overweight | 1300 |
| Being a coal miner | 1100 |
| Cancer | 980 |
| Alcohol | 130 |
| Homicide | 90 |
| Job w/ radiation exposure | 40 |
| Falls | 39 |
| Accidents to pedestrians | 37 |
| Fire (burns) | 27 |
| Generation of energy | 24 |
| Illicit drugs (U.S. average) | 18 |
| Source: Norman J. McCormick, Reliability and Risk Analysis, Academic Press, 1981, ISBN 0 | |

| TABLE D-2<br>Relative Risk of 1 in a Million Chances of Dying | |
|---|---|
| Smoking 1.4 cigarettes | Lung cancer, heart disease |
| Drinking 0.5 liter of wine | Cirrhosis of the liver |
| Eating 40 tablespoons of peanut butter | Liver cancer from aflatoxin B |
| Living 2 days in New York | Air Pollution |
| Traveling 150 miles by car | Accident |
| Flying 2500 miles by jet | Accident |
| Flying 6,000 miles by jet | Cancer from cosmic rays |
| Two month vacation in Denver | Cancer from cosmic rays |
| One chest X-ray in a good hospital | Cancer from radiation |
| Living 2 months in an average brick building | Cancer from natural radioactivity |
| Occupational radiation dose of 10 mrem | Cancer from radiation |
| Source: Wilson, R, Analyzing the Daily Risks of Life, Technology Review, 1979 | |

Note that the provenance of many of these frequently cited statistics in Table D-1 and Table D-2 are old, and may be subject to question as to what measures were actually used, and what factors incorporated, in their derivation. Many have noted problems with comparative risk analysis. Among the flaws noted (cf Merkhofer, 1987) are:

- mistrust of the communicator's motives
- the implication that risks that are small or comparable to risks that are already being accepted should themselves be accepted
- the influence of the risk measures selected, and the format in which results are presented
- scientific uncertainty in the development of the reference risks

There are several significant limitations to the use of comparative risk information: risk comparisons do not provide a simple rule for risk acceptability, the measures and format selected can easily alter the impression induced, care must be taken in the conversion of risks to common units, and the reference measures themselves are subject to the same sorts of errors and inaccuracies as the risks that are compared to it." (Merkhofer, 1987)

"All risks do not have the same characteristics and certain complexities make interpretation difficult. Comparing the risks of ingesting or inhaling an environmental contaminant to that of hang gliding, rock climbing or insect bites raises the issue of which is voluntary or involuntary. Other complexities that make comparison of risks difficult include: natural or manmade; luxury or necessity; old or new risk; catastrophic or ordinary. Because of these complexities it is seldom possible to get a good understanding of a given risk level by comparing it to other risks of the same frequency." (Cothern and Marcus, 1987)]

In spite of these difficulties, comparative risk analysis can have an important role in risk communication:

"Comparative analysis has several attractive features. It avoids the difficult and controversial task of converting diverse risks into a common monetary unit (like dollars per life lost or per case of sterlization or per day of suffering). It presents issues in a mode that is probably quite compatible with natural thought processes. Among other things, this mode may avoid any direct numerical reference to very small probabilities, for which people have little or no intuitive feel." (Merkhofer, 1987, quoting Slovic and Fischhoff, 1982, "How Safe is Safe Enough? Determinants of Perceived and Acceptable Risk" in Gould and Walker (eds.) Too Hot to Handle, Yale University Press, New Haven)

Merkhofer (1987) offers the following guidance for using comparative risk as shown in Table D-3.

**TABLE D-3**
**COMPARATIVE RISK GUIDANCE**

| Guideline | Principal Objective |
|---|---|
| Strive for neutral or value-free comparisons, make any value-laden assumptions explicit | Avoid influencing or subverting decision maker's responsibility to make value judgments |
| Use multiple comparisons based on multiple risk measures | Counter tendency of comparisons to encourage an overly simplified view of the problem |
| Tailor each comparison to illuminate a particular aspect of the risk | Increase the effectiveness of risk comparisons |
| Clarify the intent of the comparison and provide appropriate cautions | Reduce the likelihood of misinterpretation |
| Develop Reference Events iteratively with decision maker input | Increase the meaningfulness of risk comparisons |
| Explain all assumptions and uncertainties | Ensure that risk numbers attain a degree of influence commensurate with tat which they desire |

# COMMUNICATION OF QUANTITATIVE MEASURES

> "Whether it is liked or not, the decision maker needs the quantitative tool called probability and statistics." Cothern and Marcus, 1987)]

The approach to incorporating risk into decision-making recommended in this document is to make use of explicit numerical risk characterizations as part of a multiple criterion decision-making process, such that the risk associated with any given alternative is clear and is taken into account. As noted above, the display of a distribution function is the most unambiguous technical method of describing risk, but such a display is not easily incorporated into a decision-making effort, nor does it provide a good method for non-technical individuals to understand relative risks of different alternatives. Thus, simpler methods are desirable, in particular summary numerical measures and graphical displays.

> The design of displays to communicate the results of uncertainty analysis involves a number of factors. These include:
>
> - finding a clear, uncluttered graphic style and an easily understood format
> - making decisions about what information to display
> - making decisions about what information to treat in a deterministic and what to treat in a probabilistic form
> - making decisions about what kinds of parametric sensitivities will provide the key insights.
>
> (Morgan and Henrion, 1990)

## Summary Numerical Measures

When quantitative risk and uncertainty approaches are used in risk assessment, then the results should provide more than a single number. Rather, enough information should be generated by the risk assessment to provide some level of characterization of the distribution of results. The primary summary numerical measures that are used to describe, in short-hand, the characteristics of such a distribution are:

- Mean – the arithmetic average of all outcomes, the expected value.
- Standard Deviation – a measure of dispersion of a distribution, measured as a distance from the mean – the largest the standard deviation for a given mean, the more dispersed are the values.
- 95 percent Confidence Interval – A range of values in which a given parameter will fall 95 percent of the time. The width of the confidence interval is an indication of uncertainty about the given parameter.
- Maximum and Minimum

Other frequently used summary measures include:

- Median - The value for which 50 percent of the data are higher, and 50 percent are lower – the value at which the cumulative probability function is 0.5
- Modes - The observations that occur most frequently in a data set
- Quartile - When a data set, ordered from lowest to highest, is ordered and divided into four equal parts, each separating value is called a quartile (25 percent, 50 percent, 75 percent). The second quartile is the median.
- Skewness - A statistic measuring the degree of symmetry of a distribution. A plot of a skewed distribution would show a long tail to either the left or the right. Distributions with a longer upper tail are said to be positively (right) skewed, while those with a longer lower tail are negatively (left) skewed. The skewness of data is usually measured through a coefficient of skewness which is zero for symmetric distributions such as the normal or uniform distribution, is greater than zero for positively skewed data, and is less than zero for negatively skewed distributions.

Referring to the 5000 iteration dataset from the Monte Carlo simulation of Hoover Dike, the distribution of total costs was as shown (values are total cost in iteration, $ x $10^7$) in Figures D-1 and D-2:

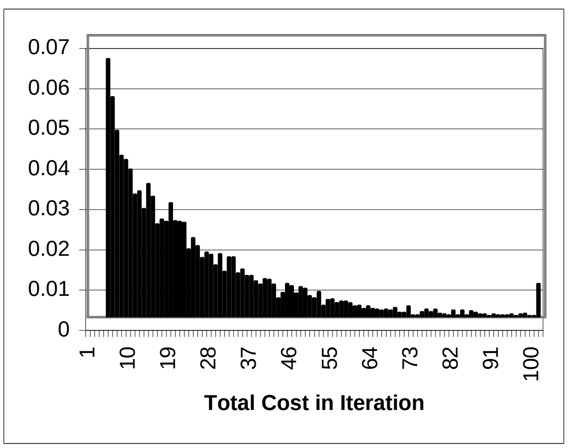


**FIGURE D-1**
**HISTOGRAM (HOOVER DIKE)**

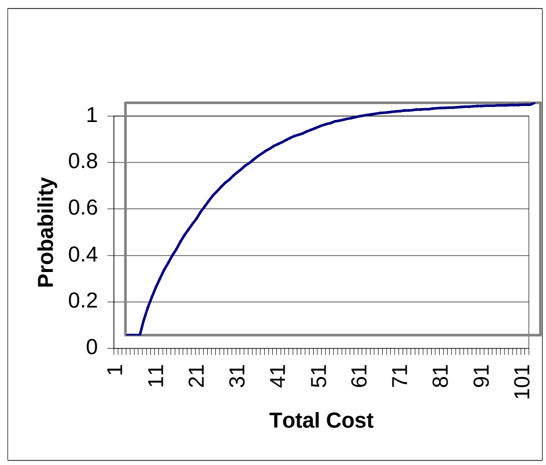


**FIGURE D-2**
**CUMULATIVE (HOOVER DIKE)**

The mean value of present value of total cost across all observations is 23.46, while the median is 17.92 (2500 observations are greater, 2500 iterations are less). The standard deviation is 19.57, a wide spread relative to the mean value. The maximum is 167.4, and the minimum is 4.1. Note that the distribution as shown in the histogram is not at all symmetric. The mode is around 4.7 (most frequent value), as shown directly from the histogram.

## Graphical Displays

### *The Distribution Function*

There are three basic methods of presenting a probability distribution over a one-dimensional uncertain quantity:

- probability density function (PDF)
- cumulative distribution function (PDF)
- Box plot

The box plot [Tukey, 1970] is a method of graphical display of elements of the distribution, constructed from summary measures, that attempts to show the most relevant graphical measures. There are many variations to the box plot. One version of the box plot makes use of the following summary statistical measures:

- minimum and maximum
- 10th and 90th percentiles
- 25th and 75th quartiles
- median

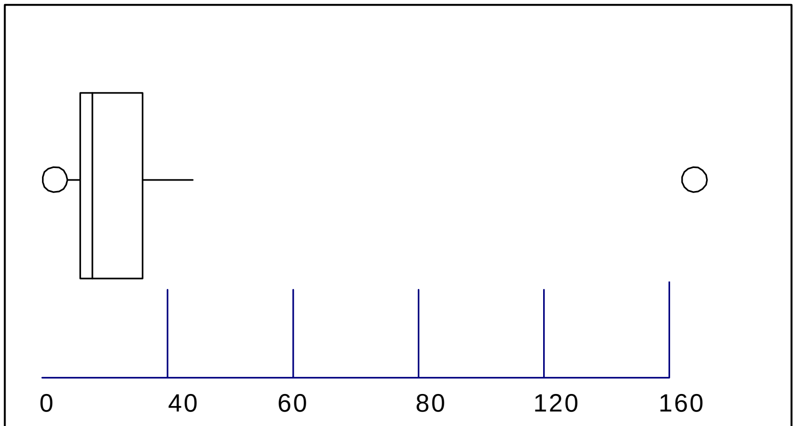


**FIGURE D-3**
**TUKEY BOX PLOT – HOOVER DIKE COST, $\$*10^7$**

and is drawn with points at the minimum and maximum, a horizontal line between the $10^{th}$ and $90^{th}$ percentiles, a box around the $25^{th}$ and $75^{th}$ quartiles, and a vertical line at the median. A sample box plot in Figure D-3, using the Hoover data, clearly shows the highly skewed nature of the distribution. One of the advantages of box plots is that, given the compact nature of the representation, a large number of individual box plots can be displayed in a single graphic, showing the ranges associated with more than a single alternative.

“Although the PDF, CDF, and box plot contain similar information, they emphasize rather different aspects of the probability distribution. ... The box plot emphasizes confidence intervals and the median. The density function clearly shows the relative probabilities of different values. It also clearly presents the mode(s) as peak(s) in the curve.” (Morgan and Henrion, 1990)

***Risk Profiles***

Examination of the distribution function is only one method of displaying the results of a risk assessment. For example, it is often important to show how one variable changes with another probabilistic variable. The following diagram, adapted from Stedinger (1996) is a simple method of showing how flood damages vary with the annual exceedance probability, under two different scenarios of dam failure. While there is no uncertainty in the damages, the plot does clearly demonstrate the impact of the two options under more extreme events. As a simple example of incorporating a display of uncertainty, the diagram shown in Figure D-4 could be revised to show the range of damages due to dam failure, for both the no action and action case, as indicated by the rectangles in Figure D-5.

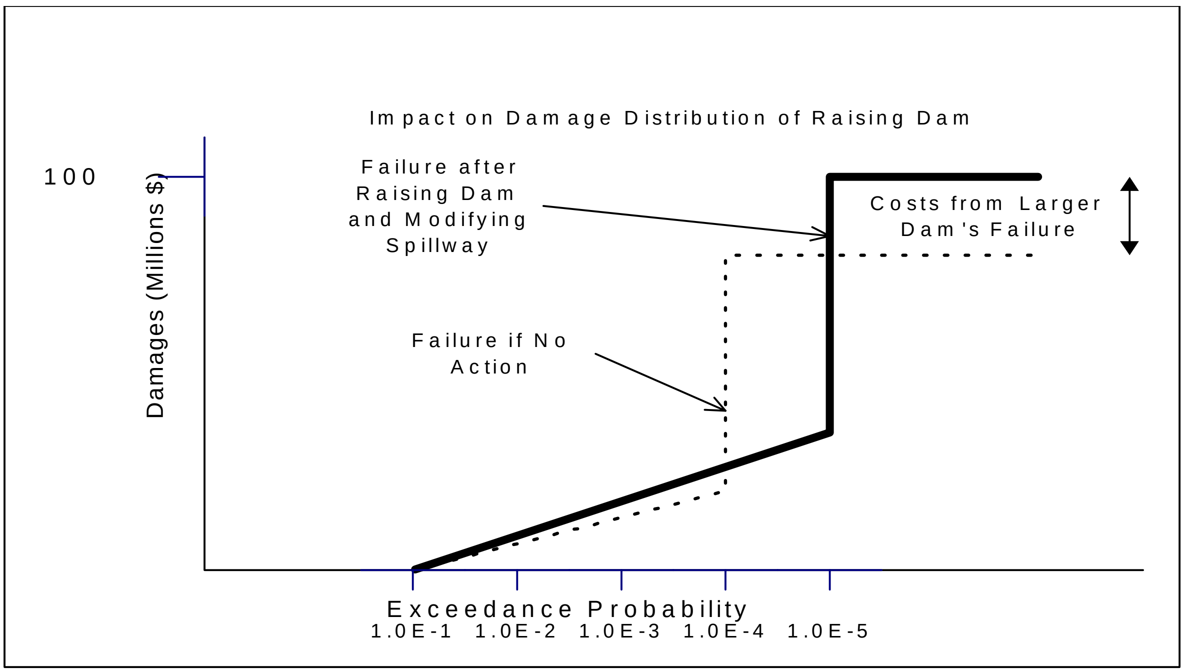


**FIGURE D-4**
**IMPACT ON DAMAGE DISTRIBUTION OF RAISING DAM**

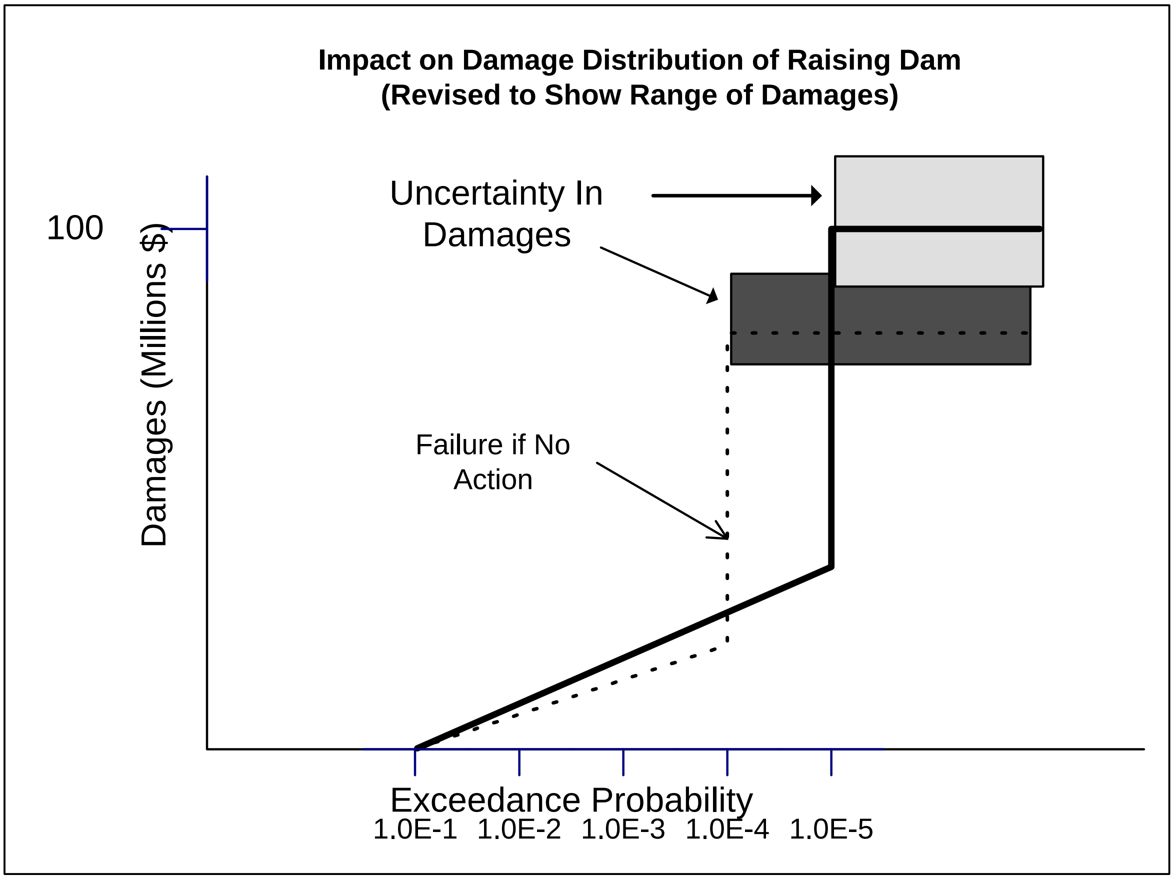


**FIGURE D-5**
**IMPACT ON DAMAGE DISTRIBUTION OF RAISING DAM**

***Expected vs. Worst Case Plot***

When a number of alternatives are being considered, and it is desired to show risk associated with choosing any alternative based on expected value, then a scatter plot showing, for each alternative, the expected value vs. the worst case value, will reveal the risk exposure. Here,"worst case" can be taken as any desired measure associated with the distribution of outcomes for each alternative, for example at the .025 probability level (lower bound of the 95 percent confidence interval), or at any other designated distance from the mean. Such information should be readily available from Monte Carlo simulations. An example of this type of display from another field is shown in Figure D-6, in which alternatives for purchasing power in the energy market, from 0 to 200 MW, are investigated, where a distribution of profits is possible for each alternative.

"The points corresponding to different power limits are connected together by a spline. The decision maker can now choose her preferred profit-risk combination from this graph. A zero power limit is obviously not optimal, because when moving from zero towards 50 MW both the expected and worst case profits increase. A risk neutral decision maker would choose the 50 MW power limit corresponding to the expected profit of 2.3. An extremely risk averse decision maker would choose the 150 MW power limit corresponding to the best worst-case profit of 1.5. In the general case the decision maker would choose some point from the Pareto-efficient frontier between the 50 and 150 MW power limits." (Makkonen and Lahdelma, 1998)

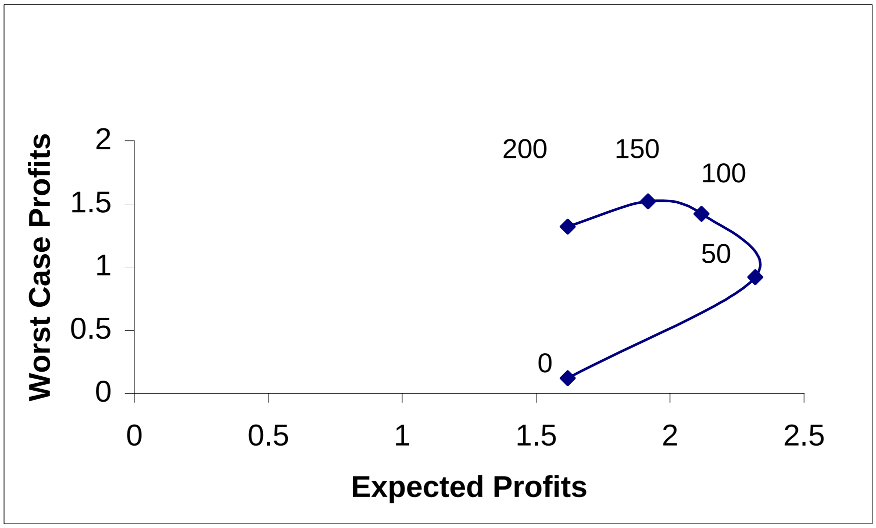


**FIGURE D-6**
**EXPECTED vs. WORST CASE PROFIT**

This simple yet powerful display can be adapted in any number of fashions, showing, for example, the best case vs. expected, or worst case vs. best-case outcomes.

### *Visualizing Uncertainty in Spatial Location*

In many cases, spatial boundaries are known only approximately (flood inundation, extent of mineral resources, etc.). A variety of attempts have been made to enhance traditional mapping techniques to more clearly demonstrate the uncertainties associated with boundary locations or the accuracy of spatial models (such as meteorological forecasting models). Cartographers have long been concerned with issues of spatial data quality, and the appropriate means of conveying uncertainties. Color, 'fuzziness' or focus, contour crispness, fill clarity, 'fog' (the transparency of the 'atmosphere' that appears to overlay a map, and resolution, have been suggested as means of displaying uncertainty in maps. The example in Figure D-7 shows how fuzzy symbols can be used to represent uncertainty in land cover. (MacEachren, 1992)

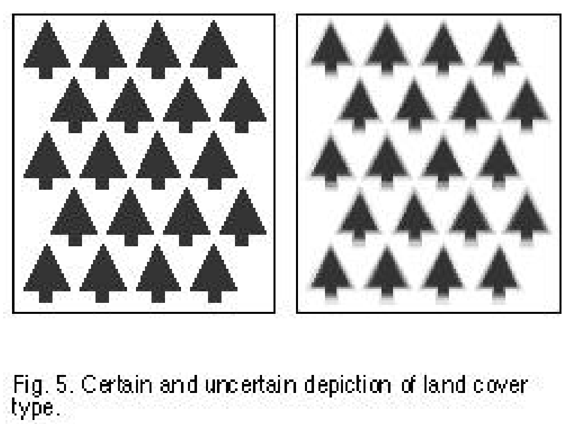


**FIGURE D-7**
**CERTAIN AND UNCERTAIN DEPICTION OF LAND COVER TYPE**

In the original of the image, presented in Figure D-8, [http://www.geovista.psu.edu/research/uncertainty/index.htm ] color is used to represent regions of uncertainty in the output from meteorological computer models. Three different forecast models are used. The graphic shows that uncertainty in the blue-to-red color fills: blue represents areas where the different forecast models agree on the predicted barometric pressure, red represents areas here the models disagree (and thus the forecast is less certain). The blue lines are isobars, lines of equal barometric pressure: they show the average pressure fields predicted by the models.

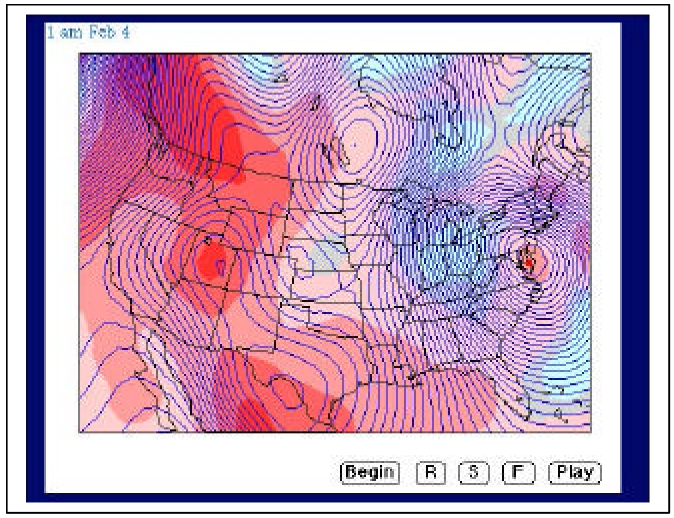


**FIGURE D-8**
**OUTPUT VISUALIZATION**

Other interesting examples can be found at:

http://www.cse.ucsc.edu/research/slvg/unvis.html.

Examples of visualizing spatial data uncertainty using animation can be found at: http://everest.hunter.cuny.edu/~chuck/CGFinal/paper.htm and http://www.geovista.psu.edu/sites/icavis/icavis/febm/sdhbivar.html

## GENERAL GUIDANCE ON VISUAL COMMUNICATION TECHNIQUES

> "It is easy now to sort through thousands of plausible varieties of graphical and statistical aggregations – and then to select for publication only those findings strongly favorable to the point of view being advocated. ... A prudent judge of evidence might well presume that those graphs, tables, and calculations revealed in a presentation are the best of all possible results chosen expressly for advancing the advocate's case" (Tufte, 1997)

Computer-based models and graphical tools make it relatively straightforward to generate a great deal of graphical data, in a variety of formats. Tufte (1983) discussed the numerous methods by which poor graphs and charts can be generated, provided principles for developing good charts, and coined the term *chartjunk* to refer to excessively decorated charts, such as those with unnecessary and misleading 3-d characteristics.

In the unpublished report "Effective Graphical Display of Water Resource Planning Information for Decision-Makers," September 1981, for the Institute for Water Resources, U.S. Army Corps of Engineers and the Office of Water Research and Technology, U.S. Department. of Interior, the authors made many of the same points as Tufte, and suggested that the relationships that are embodied in the graphic be explicitly stated. That is, the creator of the visualization method should decide what message or relationship the graphic should communicate, and craft the visualization to emphasize that message (without distorting or misleading). In general, graphical presentation is not sufficient for communication—the graph must be designed to highlight a particular point to be made, and the point must as well be stated. This is in contrast to an approach in which data are simply presented, and the audience is left to figure out what the data mean. This does not imply that graphics should be inaccurate or misleading – rather, they should be designed to emphasize the point that has already been developed by the author. Further, the creator should be aware of the techniques normally used by the audience in interpretation, and not place too high a demand on the audience to learn a new technique.

Good graphics are the result of:

- Simplicity of presentation
- Style Consistency
- Suitability of Technique
- Significance of Message
- Stated Relationship
- Strengthened Relationship

Evaluating a graphic:

- Does it communicate something?
- Does it use good practice?
- Does it distort?
- Is it attractive? appropriate? necessary? helpful?
- Is clarity sacrificed to 'design' or 'artistic' values?
- Is the relationship shown in the graphic also explicitly stated as text? (Does it tell you what you should see?)
- Is the relationship strengthened and emphasized?
- Is the graphic simple and easy to understand?
- Is it worth the trouble it took to create it?

For the specific situation of communicating about uncertainty, the following are typical needs:

- display of uncertainty about a single parameter (in a single alternative)
- Use a display of the probability density function and cumulative density function or a simplification such as a box plot.
- display of change in uncertainty about a single parameter over time
- plot of variable against time , showing error bands
- demonstration of how uncertainty in one or more parameters affects the uncertainty of other parameters
- schematic influence diagram
- display of uncertainty vs. expected value for a number of alternatives
- multiple Box Plots

- expected value vs. worst case plot
- display of spatial uncertainty (uncertainty in boundaries)
- ‘fuzzy’ mapping techniques, use of color